

Thank you for taking part in marking the scripts. It is of enormous benefit to young students to be able to take part in these competitions, tackling much harder problems than they normally get, and be able to have them marked by physicists who know what they are doing. It is because of your expertise in the subject that it is possible to do this. Exams marked by non-specialists are more of a tick box exercise, which is of little value in stretching students to grasp the subject at a deeper level. The layout of the work may be annoying. That is a national problem and we are not going to change that easily, although we do our best, as do the teachers of these students.

- We need **ACCURACY** and **CONSISTENCY from you**. Care is required. The mark scheme has marks allocated to make the marking easier. The mark allocation is different to the paper. **USE THE MARK SCHEME ALLOCATIONS.**
- You do not need to spend time working through laborious arithmetic calculations. The marks are set out for steps achieved. **Positive marking – if almost there then give the mark. Look for the opportunity to award the mark, but do not be careless about this.**
- **Full marks are awarded for the correct answer, provided that there is some supporting working** and it is not a “show that” question. Look out for suspicious results with **insignificant working** but no need to read in detail. You are NOT required to spend time deciphering **scribble**.
- **Positive marking** is the aim. Marks should be awarded for good physics, even if the reasoning does not follow the mark scheme. Alternative routes to the answers can be allowed.
- How to mark – ticks for marks only, not for your notation. You may write on script, make comments. They are not returned, but we will see them. We do checks on papers.
- Enter the Total for Section 1 questions.
- **Significant figures**. This is not a test of significant figures. A leeway of  $\pm 1$  sig. fig. is generally allowed, but sometimes it is just a close numerical result indicating they have got the physics right. Some answers can be left in fractional form.
- **Units** should be given for the final answer. It may be that the unit is given a little earlier and that it does not appear on the very last line. Allowance may be made if it is clear that the unit has been used a line or two earlier.
- If the units are a required part of the answer for the mark, they must be there.
- **Error carried forward** (ecf) is allowed provided ridiculous results do not start appearing. A mark is lost for the initial mistake, but then they can carry on (if it is possible) to gain some of the subsequent marks for the next one or two steps only. Just make a decision as to whether they should have the single mark or not.
- There may be a lot of working for the answer. If they are almost there, you may give the mark even if there is a numerical mistake in the last line. Use your judgement. The ticks for the marks are not exact i.e. they are for the idea and almost getting there.
- You must follow the mark scheme so that we mark **CONSISTENTLY**. Do not make your own independent mark scheme. Avoid relying on your memory for the mark allocation. You need the mark scheme open beside you.

**If you need advice, email Robin Hughes [rh584@cam.ac.uk](mailto:rh584@cam.ac.uk) . I will respond promptly.**

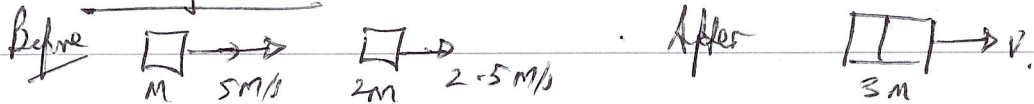
**You can send a phone photo or just ask a question.**

1. **Do not mix up students' names when entering marks.**
2. **Add up the marks correctly.** Check your addition. For each little section, note the total in a circle, as in the mark scheme. Then note the total for the page at the bottom in two parallel lines to distinguish it.
3. Do not leave “pdf papers” in any unsecured place. They are confidential.
4. You can write on the answers if it helps you keep track. Ticks are for marks so do not tick everything. Ticks are counted up when checking.
5. Updates to the mark scheme are inevitable and will be sent out. If you see a mistake, please email me.

Section 1 Question 1 Nov 2021

1

(a) Colliding trucks



Momentum Conservation:

$$5 \times m + 2.5 \times 2m = 10m = 3mV$$

final speed,  $V = \frac{10}{3} = \underline{\underline{3.3 \text{ m/s}}}$  ✓

Initial KE =  $\frac{1}{2} \cdot m \cdot 5^2 + \frac{1}{2} \cdot 2m \cdot 2.5^2$   
 $= \left( \frac{25}{2} + \frac{25}{4} \right) m = \underline{\underline{\frac{75}{4} m}}$  ✓

Final KE =  $\frac{1}{2} \cdot 3m \cdot \left( \frac{10}{3} \right)^2$   
 $= \frac{1}{2} \cdot 3m \cdot \frac{100}{9}$   
 $= \underline{\underline{\frac{50}{3} m}}$

Loss of KE =  $\frac{75}{4} m - \frac{50}{3} m = \underline{\underline{\frac{25}{12} m}}$

percentage loss =  $\frac{25 \text{ m}}{12} \div \frac{75 \text{ m}}{4} = \frac{1}{9} = \underline{\underline{11\%}}$  ✓

3

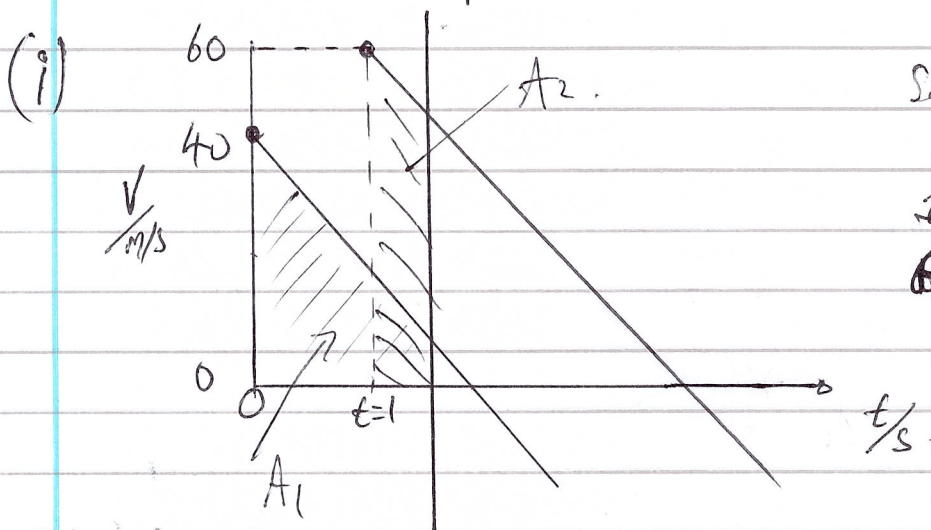
(b) Drum skin

$[p] = 0$  or no units, just a number, dimensionless. ✓  
 $[f] = \frac{[k][u]}{[a^2]} \quad T^{-1} = \frac{L \cdot L T^{-2}}{[a^2]}$

$[a] = L = \text{metres}$   
 $\uparrow \quad \uparrow \text{either}$

2

(c) Ball thrown upwards



Sample sketch  
parallel slopes.  
2 second offset.  
60 below horizontal axis ✓

(ii) They meet when  $A_1 = A_2$ .

$A_1$  is  $S_{40} = 40t - \frac{1}{2} \cdot 10 \cdot t^2$

$A_2$  is  $S_{60} = 60(t-1) - \frac{1}{2} \cdot 10(t-1)^2$

When,  $S_{40} = S_{60}$

then  $40t - 5t^2 = 60t - 60 - 5(t^2 + 1 - 2t)$

$0 = 20t - 60 - 5 + 10t$

$t = \frac{65}{30} = \frac{13}{6} \text{ s} \approx 2.17 \text{ s}$

} equal area  
at equation ✓

$S_{40}$  is 40 m/s ball  
 $S_{60}$  is 60 m/s ball.

If  $g$  used, then  $t = \frac{60 + g}{20 + g} = 2.34 \text{ s}$

} any to 2sf  
[1.3s, 2.2s, 2.3s] ✓

$S_{40} = 40 \cdot \frac{13}{6} - \frac{g}{2} \cdot \frac{169}{36}$   
 $= \frac{13}{6} (40 - \frac{5 \cdot 13}{6}) = 63.2 \text{ m}$  (2sf)

} either ✓

If  $g = 9.81$  used,  $S_{40} = 66.8 \text{ m}$  (2sf)

(iii)  $t = 4$  seconds,  $S = 80 \text{ m}$

time for  $S_{60}$  to reach 80m:  $v^2 = u^2 - 2gh$   
 $= 60^2 - 2 \cdot 10 \cdot 80$   
 $= 2000 \frac{\text{m}^2}{\text{s}^2}$   
 $v = 20\sqrt{5} \text{ m/s}$

$t = \frac{u-v}{g} = \frac{60 - 20\sqrt{5}}{10} = 6 - 2\sqrt{5}$

$\Delta t = \frac{g}{g} - (6 - 2\sqrt{5})$  either  
 $= 2.47 \text{ s}$  (2sf)

✓ 2.5s

If  $g = 9.81$  used,  $\Delta t = \frac{20(\sqrt{5}-1)}{9} = 2.52 \text{ s}$  (2sf)

5

(d) Railway Carriage with liquid.

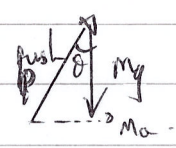
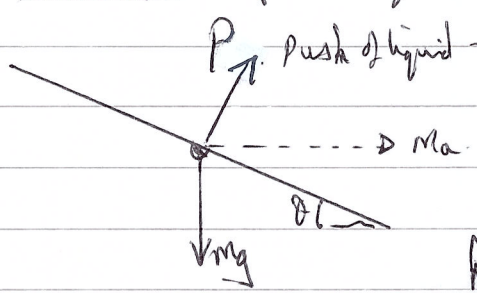


Diagram of some kind ✓

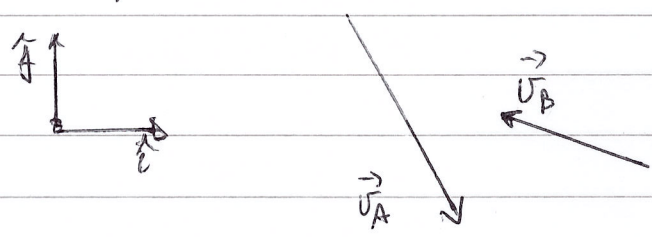
Resolve V:  $P \cos \theta = mg$  ✓  
 H:  $P \sin \theta = ma$

$\tan \theta = \frac{a}{g} = \frac{0.84}{9.81}$

$\theta = 4.9 \approx 5^\circ$  ✓

(e) Aeroplanes.

3



(i) Subtract  $\vec{u}_B$  from  $\vec{u}_A$ :  $\vec{u}'_A = (50+90)\hat{i} + (-125-60)\hat{j}$   
 $\vec{u}'_A = 140\hat{i} - 185\hat{j}$  ✓

(ii) At  $t=0$ , position of A with B at the origin is  $-400\hat{i} + 1200\hat{j}$

( $\vec{r}'_A$  is in next frame of B)  $\vec{r}'_A = -1200\hat{i} + 1800\hat{j}$  with B at (0,0)

Some or a constructive approach! ✓

As  $t$  increases  $\vec{r}'_A(t) = -1200\hat{i} + 140\hat{i}t + 1800\hat{j} - 185\hat{j}t$   
 $= (-1200 + 140t)\hat{i} + (1800 - 185t)\hat{j}$

Distance from origin  $D = \sqrt{x^2 + y^2}$  so,  
 $D^2 = (-1200 + 140t)^2 + (1800 - 185t)^2$

to find minimum  $\frac{dD^2}{dt} = 2(-1200 + 140t)140 + 2(1800 - 185t)(-185)$   
 $0 = -1200 \times 140 + 140^2 t - 1800 \times 185 + 185^2 t$   
 $t = 9.31 \text{ s}$  ✓

So  $D^2 = (-1200 + 140 \times 9.31)^2 + (1800 - 185 \times 9.31)^2$   
 $D = 129 \text{ m} = 130 \text{ m. (2sf)}$  ✓

4

(f) Water flow

energy from supply = thermal energy gained by water  
 $\frac{230^2}{R} \Delta t = M C \Delta T$  ✓

$$\frac{230^2}{R} = \frac{M}{\Delta t} \cdot C \cdot \Delta T$$

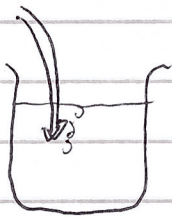
$$\frac{230^2}{R} = \rho \frac{\Delta \text{Vol}}{\Delta t} \cdot C \cdot \Delta T$$

conversion of  $\frac{M}{\Delta t}$  ✓

$$\frac{230^2}{R} = 1000 \times \left(\frac{10^{-3}}{60}\right) \times 4180 \times 60$$

$$R = \frac{230^2}{\frac{4180}{13.5}} = 12.7 \Omega \quad (2 \text{ sf}) \quad \checkmark \quad \boxed{3}$$

(g) Dry Steam



loss of energy of steam + loss of energy of condensed water = energy gained by cold water + calorimeter ✓ idea

$$M_s h + (100 - 30) C_w \cdot M_s = M_w \cdot C_w \cdot (30 - 0)$$

✓ equation

$$M_s (L + 70 C_w) = M_w \cdot C_w \cdot 30$$

$$L = \frac{M_w \cdot 30 \cdot C_w}{M_s} - 70 C_w$$

$$= \left(\frac{M_w}{M_s} \cdot 30 - 70\right) C_w$$

calorimeter included ✓

$$= \left(\frac{250 + 10}{12.8} \cdot 30 - 70\right) 4180$$

$$= 2.25 \frac{\text{MJ}}{\text{kg}} \quad (2 \text{ sf}) \quad \checkmark \quad \boxed{4}$$

(h) Expansion

$$(l_{Fe} - l_{0Fe}) = l_{0Fe} \cdot \alpha_{Fe} \Delta T$$

and  $(l_{Cu} - l_{0Cu}) = l_{0Cu} \alpha_{Cu} \Delta T$  ✓



expansions are same, for  $\Delta T$ .

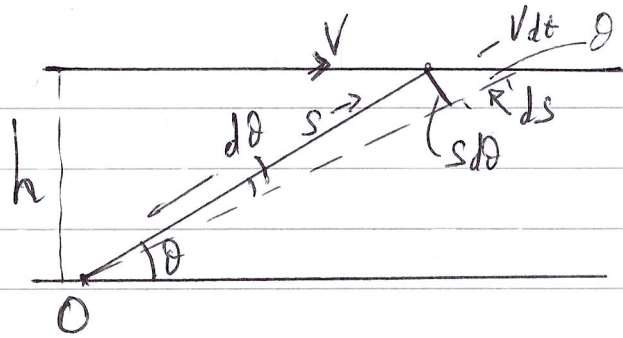
$$(l - l_0) \text{ so } l_{0Fe} \cdot \alpha_{Fe} = l_{0Cu} \alpha_{Cu} \quad \checkmark$$

$$\frac{l_{0Cu}}{l_{0Fe}} = \frac{\alpha_{Fe}}{\alpha_{Cu}}$$

$$l_{0Cu} = \frac{11.9}{17} = 0.70 \text{ m} = 70 \text{ cm} \quad (2 \text{ sf}) \quad \checkmark \quad \boxed{3}$$

5

(i) Steeplane



✓ diagram.

$\theta = 60^\circ$

$h = 3000\text{m}$

(i) distance from base to top,  $s = \frac{h}{\sin 60^\circ} = 3464 = \underline{\underline{3500\text{ m}}}$  ✓

(ii) from diagram  $\frac{s d\theta}{v dt} = \sin \theta \Rightarrow \frac{d\theta}{dt} = \frac{v \sin \theta}{s} = \frac{v h}{s^2}$  from expression equal to  $\frac{d\theta}{dt}$ .

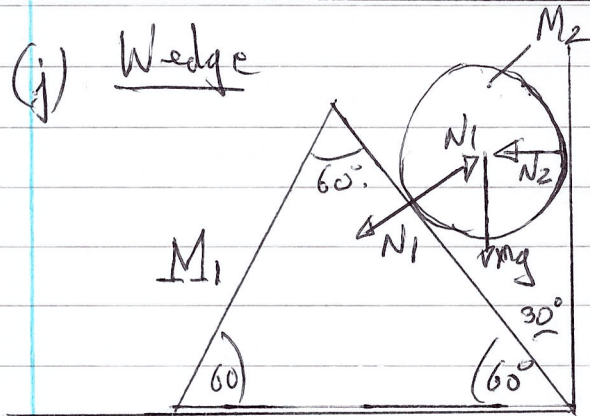
So  $v = \frac{s^2 d\theta}{h dt} = \frac{(3464)^2 \times 0.009}{3000}$

$v = 360\text{ m/s}$  ✓

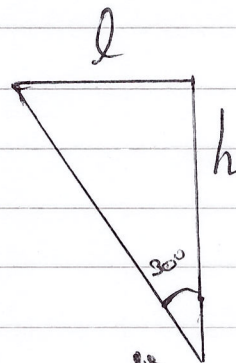
(ii)  $\frac{ds}{v dt} = \cos \theta \Rightarrow \frac{ds}{dt} = v \cdot \omega \theta = \frac{v}{2} = \underline{\underline{180\text{ m/s}}}$  ✓

5

(j) Wedge



(i)



$\tan 30^\circ = \frac{h}{\sqrt{3}l} = \frac{l}{h}$

$h = \sqrt{3}l$  ✓

So  $h = a_{\text{sphere}} = \sqrt{3} a_{\text{wedge}} = \sqrt{3}l$  ✓

(ii) On wedge resolve H:  $N_1 \cos 30^\circ = M_1 a_{\text{wedge}}$

On sphere:  $m_2 g - N_1 \sin 30^\circ = M_2 a_{\text{sphere}}$

Hence  $M_2 g - M_1 \frac{a_{\text{sphere}} \cdot \sin 30^\circ}{\sqrt{3} \cos 30^\circ} = M_2 \cdot a_{\text{sphere}}$  ✓

$M_2 g - M_1 a_{\text{sphere}} = M_2 a_{\text{sphere}}$

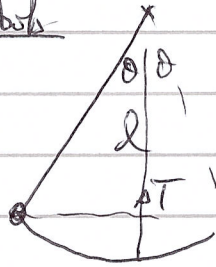
$3 M_2 g = 3 (M_1 + 3 M_2) a_{\text{sphere}}$

$a_{\text{sphere}} = \frac{3 M_2 g}{(M_1 + 3 M_2)}$  ✓

5

6

(K) Pendulum bob



T is due to the weight + it provides the centripetal force required.

$$T = mg + \frac{mv^2}{l}$$

Energy:  $mg(l - l \cos \theta) = \frac{1}{2}mv^2$

$$2gl(1 - \cos \theta) = v^2$$

$$\text{So } T = mg + 2mg(1 - \cos \theta)$$

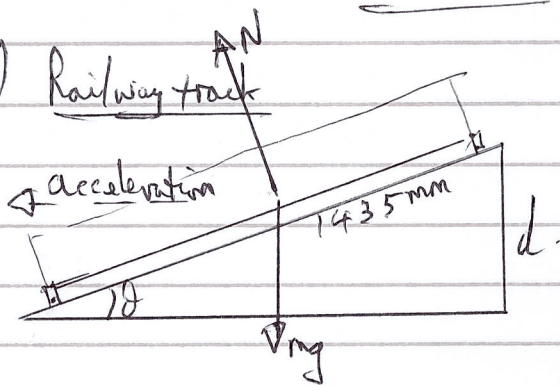
$$\text{ie } nmg = mg + 2mg - 2mg \cos \theta$$

$$n = 3 - 2 \cos \theta$$

$$\cos \theta = \frac{1}{2}(3 - n)$$

4

(L) Railway track



require no vertical component of acceleration.

Diagram or requirement for "acceleration inwards"

$$\text{resolve } H: N \sin \theta = \frac{mv^2}{r}$$

$$N \cos \theta = mg$$

$$\tan \theta = \frac{v^2}{rg}$$

$$= \frac{(200 \times 10^3)^2}{3.6 \times 10^3 \times 1500 \times 9.8}$$

$$= 0.2097$$

$$\theta = 11.85^\circ$$

$$d = 1435 \times \sin \theta$$

$$= \underline{295 \text{ mm}}$$

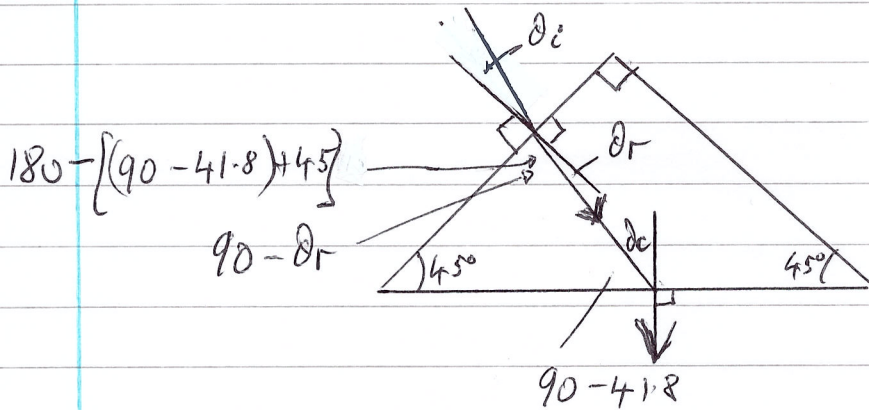
5

(M) Prism

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$1.5 \sin \theta_c = 1 \sin 90$$

$$\sin \theta_c = \frac{2}{3} \rightarrow \theta_c = \underline{41.8^\circ}$$



✓ correct diagram

$$\therefore 180 - [(90 - 41.8) + 45] = 90 - \theta_r$$

$$\theta_r = 3.2^\circ$$

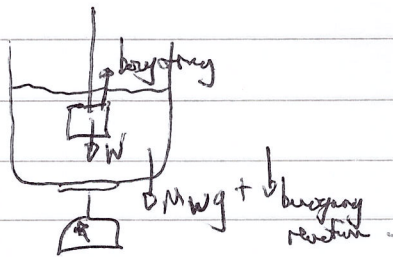
$$\sin \theta_i = 1.5 \sin \theta_r$$

$$\theta_i = \underline{4.8^\circ}$$



3

(N) Aluminium block



Weight of block.  $MA \cdot g = \rho_A \cdot V \cdot g$   
 upthrust is  $(\rho_w V) g$  ✓

$$\therefore \rho_A = 2.69 \frac{g}{cm^3}$$

✓ (or it's correct appearance in equation)

Reading on the newton-meter is  $(\rho_A - \rho_w) V g$

$$= \rho_A \cdot V \cdot g \left(1 - \frac{\rho_w}{\rho_A}\right)$$

$$= 6.6 \left(1 - \frac{\rho_w}{\rho_A}\right)$$

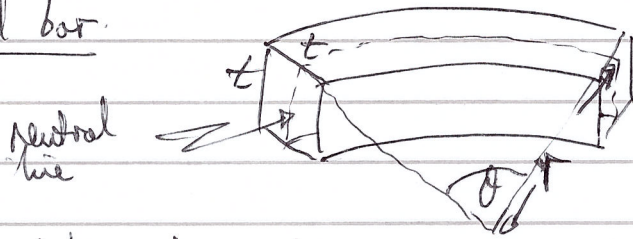
$$= \underline{4.15 \text{ N}} \quad (2 \text{ sf}) \quad \checkmark$$

Balance reads  $600 \text{ g} + \rho_w \cdot V$   
 $= 600 + 1.0 \times 250$   
 $= \underline{850 \text{ g}} \quad (2 \text{ sf}) \quad \checkmark$

4

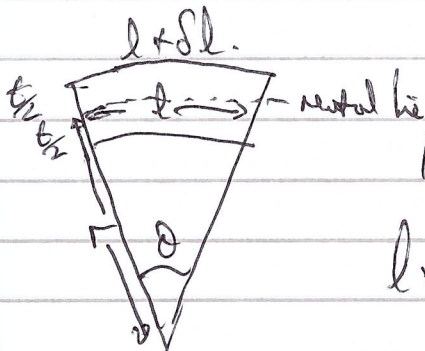


(c) Steel bar.



The steel bar is elastic and compresses as easily as it stretches, like a spring.  
 But it will break under tension.

If it is slightly stretched and slightly compressed by bending, there will be a neutral line close to the centre line, if the bending is small.



Using  $\theta$  in radians.

Diagram ✓

$$l + \delta l = (r + \frac{t}{2}) \theta$$

$$l = r \theta \text{ (along neutral line) } \checkmark$$

$$\text{So } \delta l = \frac{t}{2} \theta$$

$$\text{and hence } \frac{\delta l}{l} = \frac{t \theta}{2 r \theta} = \frac{t}{2r} \checkmark$$

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\frac{\delta l}{l}} = \frac{\sigma l}{\delta l}$$

$$Y = 210 \times 10^9 \text{ Pa} \quad Y = \frac{\sigma 2r}{t}$$

$$\sigma_{\text{breaking}} = 840 \times 10^6 \text{ Pa}$$

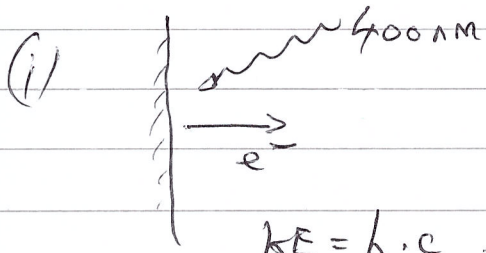
$$\text{So } 210 \times 10^9 = 840 \times 10^6 \times \frac{2 \times r}{0.02}$$

$$r = \frac{210 \times 10^9}{840 \times 10^6} \times \frac{0.02}{2}$$

$$r = 2.5 \text{ m} \checkmark$$

4

(p) photoelectric effect



$$\begin{aligned}
 KE &= h \cdot \frac{c}{\lambda} - 2.28 \text{ eV} \quad \checkmark \text{ (in some form)} \\
 &= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{400 \times 10^{-9}} - 2.28 \text{ eV} \\
 &= 3.108 - 2.28 \\
 &= 0.828 \text{ eV} \\
 &= 1.325 \times 10^{-19} \text{ J} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{ eV for } \checkmark
 \end{aligned}$$

Now  $v = \sqrt{\frac{2 \times KE}{m}}$   $\checkmark$

$$= \sqrt{\frac{2 \times 1.325 \times 10^{-19}}{9.11 \times 10^{-31}}}$$

$$= 5.39 \times 10^5 \text{ m/s}$$

So  $t = \frac{l}{v} = \underline{1.85 \times 10^{-7} \text{ s}}$   $\checkmark$  (2.5)

(ii) Electron will lose 0.5 eV of KE in the electric field  
 So has KE of  $K' = 0.828 - 0.5 \text{ eV}$   
 $= 0.328 \text{ eV}$ .

So  $v_{\text{final}} = \sqrt{\frac{2K'}{m}} = 3.39 \times 10^5 \text{ m/s}$   $\checkmark$

In a uniform E field, the force and acceleration are constant  
 So average speed of  $5.39 \times 10^5$  and  $3.39 \times 10^5 \text{ m/s}$   
 is  $4.39 \times 10^5 \text{ m/s}$

Area  $\Delta t = \frac{0.1}{4.39 \times 10^5} = 2.28 \times 10^{-7} \text{ s}$   $\checkmark$

(9) Balanced circuit: (10)  
 If we adjust R, we can balance the potentials across 10Ω so that they are equal.  
 To obtain zero potential across 10Ω resistor ✓ or balance idea.

$$\frac{(4+2)}{(R//20)} = \frac{8}{24}$$

ratio ✓

$$\frac{6}{\frac{R \cdot 20}{R+20}} = \frac{1}{3} \rightarrow 3 \cdot 6 = \frac{20 \cdot R}{R+20}$$

$$18(R+20) = 20R$$

$$18R + 360 = 20R$$

$$\underline{\underline{R = 180 \Omega}}$$

✓ 3

(10) Charge on a capacitor

Capacitors in parallel.  $2 + 4 = 6 \mu F$  ✓

Then in series

$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{5}$$

$$C_{eq} = \frac{30}{11} \mu F$$
 ✓

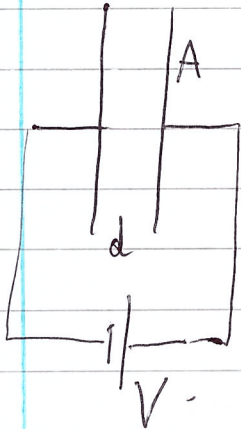
$$Q \text{ which flows} = V C_{eq} = 9 \times \frac{30}{11} = \frac{270}{11} \mu C$$

$$\text{and the share on } C_4 \text{ is } \frac{2}{3} \times \frac{270}{11} = \frac{180}{11} = 16.4 \mu C \checkmark$$

(2.4)

3

(S) parallel plate capacitor



$E_1 = \frac{1}{2} CV^2$ , but C changes, so express in geometric factors  
 $E_1 = \frac{\epsilon_0 A V^2}{2d}$  in terms of  $V^2$  ✓

V is constant,  $d \rightarrow \frac{d}{3}$

So  $E_2 = 3 E_1$  ✓

Then  $E_2 = \frac{Q^2}{2C} = \frac{Q^2 \cdot d_2}{2\epsilon_0 A}$  in terms of  $Q^2$  ✓

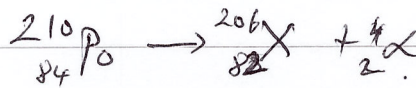
Q is fixed as the battery is disconnected  
 $d_2 \rightarrow 3d$

$E_3 = 3 E_2 = 9 E_1$

So,  $E_3 - E_1 = 8 E_1$   
 $= 4 \frac{\epsilon_0 A V^2}{d}$  ✓

4

(t) Radioactive decay



exponential factors ✓

Number remaining after 100 days is  $N = N_0 e^{-\lambda t}$   
 $= N_0 e^{-\frac{100 \times 0.693}{138}}$

$= N_0 \times 0.587$  ✓  $(\approx \frac{N_0}{\sqrt{e}})$

number of  $\text{Po}$  lost is  $N_0 - N$   
 $= N_0 (1 - 0.587)$   
 $= N_0 \cdot 0.413$  ✓

This is the number of  $\alpha$  produced.

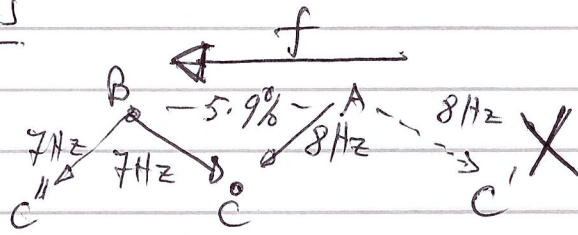
No of moles at start is  $\frac{5 \times 10^{-3} \text{ g}}{210 \text{ g}} = 2.38 \times 10^{-5}$  ✓

Hence mass of alpha produced =  $0.413 \times 2.38 \times 10^{-5} \times 4 \text{ g}$  ✓  
 $= 3.9 \times 10^{-5} \text{ g}$  ✓  
fraction mass of a mole

which is the mass loss of polonium

6

(U) Beats

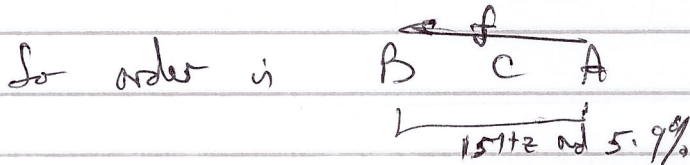


If C' is below A, then B would need to be below A, which is not correct.

If C' is above B, then BA is  $(8-7) \text{ Hz} = 1 \text{ Hz}$

But if AB is 1 Hz and 5.9% different, then A is 17 Hz ✓  
which is below acceptable.

$$\left[ \begin{aligned} f_B &= f_A(1 + 0.059) \text{ and } f_B - f_A = 1 \text{ Hz} \\ \rightarrow 1 + f_A &= f_A + 0.059 f_A \\ f_A &= \frac{1}{0.059} = 17 \text{ Hz} \end{aligned} \right].$$



So

A is 254.2 Hz	} any two ✓✓
B is 269.2 Hz	
C is 262.2 Hz	

Correct order  $A < C < B$  ✓

4

(V) Resistance of a filament light bulb

given  $R = A + BP$ .

$$\text{and } P = \frac{V^2}{R} \text{ so } R = \frac{V^2}{P} = \frac{230^2}{100} = 23^2 R.$$

$$\text{Then } 23^2 = A + B \cdot 100 \quad \textcircled{1} \quad \checkmark$$

$$\text{Also, given } \frac{23^2}{5} = A + B \cdot 500 \quad \textcircled{2}$$

$$\text{Subtracting } 23^2 \left(1 - \frac{1}{5}\right) = B \cdot 100 (1-5)$$

$$\Rightarrow B = \frac{-23^2}{500} = -1.058 \frac{R}{W} \quad \checkmark$$

$$\text{Then } A = \frac{6 \cdot 23^2}{5} = 634.8 R. \quad \checkmark$$

From  $R = A + BP$  we can substitute for  $R$  with  $\frac{V^2}{P}$

$$\text{Then } \frac{V^2}{P} = A + BP \rightarrow V^2 = AP + BP^2 \quad \text{quadratic } \checkmark$$

$$\text{or } BP^2 + AP - V^2 = 0$$

$$\text{solving, } P = \frac{-A \pm \sqrt{A^2 + 4BV^2}}{2B}$$

substituting values,  $V = 210 V, A, B.$

$$\text{then } P = 80 W \quad \checkmark$$

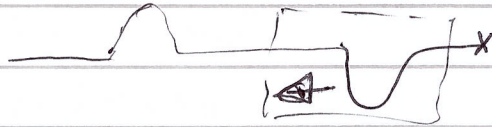
5

## SECTION 2

(14)

Q2

(a)(i)



✓

(ii)

$$t = \frac{2l}{v}$$

✓

(iii)

$$f_1 = \frac{v}{2l}$$

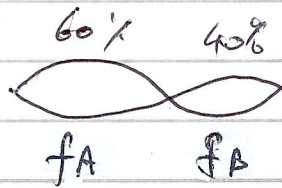
✓

(iv)

$$f_n = \frac{nv}{2l}$$

✓

(v)



$$f_A = \frac{v}{2 \times 0.6l}$$

$$= \frac{v}{1.2l}$$

$$= \frac{2f_1}{1.2} = \underline{\underline{1.7f_1}}$$

$$f_B = \frac{v}{2 \times 0.4l}$$

$$= \frac{v}{0.8l} = \frac{v}{0.8l} = 2.5f_1 = \underline{\underline{2.5f_1}}$$

$$= \underline{\underline{\frac{5}{2}f_1}}$$

✓ one OR both correct.

(vi)

$$T = \mu v^2 = \mu f^2 \lambda^2 = \mu f^2 \left(\frac{2l}{5}\right)^2 = \mu f^2 \frac{4}{25} l^2$$

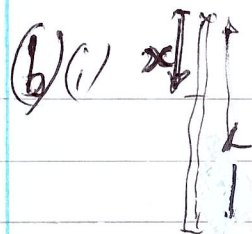
$$= \frac{4.2 \times 10^{-3}}{2.3} \times 162^2 \times \frac{4}{25} \times (2.3)^2 = 405.6 \text{ N}$$

$$= \underline{\underline{410 \text{ N}}}$$

fundamental harmonic:  $l = 5 \times \frac{\lambda}{2} \Rightarrow \lambda = \underline{\underline{\frac{2}{5}l}}$

✓

16



$$T = (L-x) \frac{M}{L} g$$

✓

(ii)

$$T_b = (L - \frac{3}{4}L) \frac{M}{2} g$$

$$T_b = \frac{M}{4} g$$

$$v_b = \sqrt{\frac{Mg}{\mu} \cdot \frac{1}{2}}$$

$$T_t = (L - \frac{1}{4}L) \frac{M}{2} g = \frac{3}{4} Mg$$

$$v_t = \sqrt{\frac{Mg}{\mu} \cdot \frac{\sqrt{3}}{2}}$$

either correct ✓

$$\frac{v_t}{v_b} = \sqrt{3} \quad \text{or} \quad \frac{v_b}{v_t} = \frac{1}{\sqrt{3}}$$

(iii)

$$v = \sqrt{\frac{(L-x) \frac{M}{L} g}{\mu}} = \sqrt{(L-x)g}$$

(iv)

$$t_{top} = \int_0^x \frac{dx}{v} = \frac{1}{\sqrt{g}} \int_0^x \frac{dx}{\sqrt{L-x}}$$

and integrating,

$$t_{top} = 2 \sqrt{\frac{L}{g}} \left( 1 - \sqrt{1 - \frac{x}{L}} \right)$$

(v)

falling ball:  $s = \frac{1}{2} g t^2$  so,  $t = \sqrt{\frac{2h}{g}}$

In our case, we require  $\sqrt{\frac{2x}{g}} = 2 \sqrt{\frac{L}{g}} \left( 1 - \sqrt{1 - \frac{x}{L}} \right)$

So,  $\frac{2x}{g} = \frac{4L}{g} \left( 1 - \sqrt{1 - \frac{x}{L}} \right)^2 \rightarrow \frac{x}{2L} = 1 + 1 - \frac{x}{L} - 2 \sqrt{1 - \frac{x}{L}}$

Rearranging, squaring etc.

$$x = \frac{8}{9} L$$

8



(C) pulse timing time time  $t_1, t_2$  are the absolute time  
 $T_W$   $t_1$   $\times$   $t_2$   $T_E$

$$c t_1 + c t_2 = 6 \times 10^5 \text{ m}$$

and  $t_2 - t_1 = \Delta t = (1.542 \times 10^{-3} \text{ s} - 0.484 \times 10^{-3} \text{ s})$   
 $= 1.058 \times 10^{-3} \text{ s}$  ✓  
 and using  $c = 3 \times 10^8 \text{ m/s}$   
 $t_2 + t_1 = 2 \times 10^{-3} \text{ s}$

$$\text{So } 2t_1 = 2 \times 10^{-3} - 1.058 \times 10^{-3}$$

$$t_1 = 0.471 \times 10^{-3} \text{ s}$$

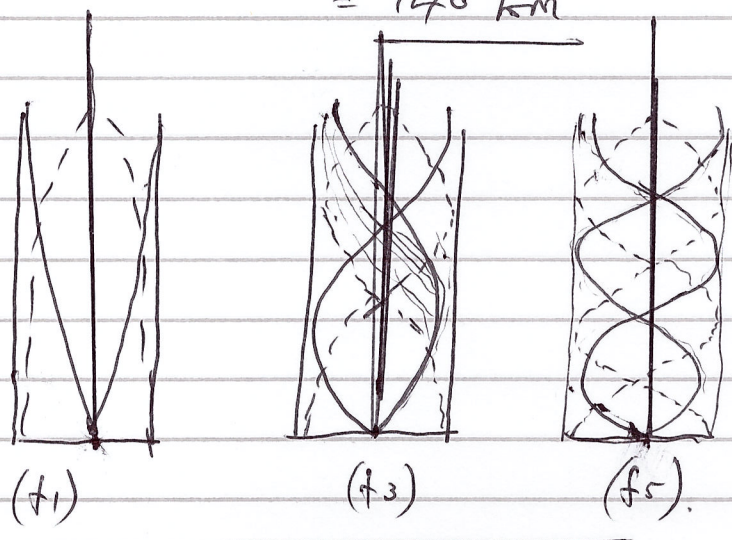
$$c t_1 = 141.3 \text{ km}$$

$$= 140 \text{ km}$$

✓ [2]

(d)

- - - pressure  
 — amplitude



amplitude ✓  
 pressure ✓

[2]

(e) (i)  $\Delta P = \rho g \Delta h$   
 $= 1.225 \times 9.81 \times 20$   
 $= 240 \text{ Pa}$   
 $\frac{\Delta P}{P} = 0.24 \%$  ✓

(ii)  $P \propto \rho$  so the ratio is constant. ✓  
 (no other relevant factors)

(iii)  $v = \sqrt{\frac{\gamma P}{\rho}}$  and  $PV = nRT$   
 also  $M_\mu = \frac{[kg]}{[mol]} = \frac{M}{n} = \frac{\rho V}{n} \Rightarrow \frac{n}{V} = \frac{\rho}{M_\mu}$

Hence  $v = \sqrt{\frac{\gamma RT}{M_\mu}}$  ✓  
 $v \propto \sqrt{T}$  ✓ this is the mark.

C = cold H = hot.

(17)

(iv)  $l_{1+\epsilon} = \frac{\lambda_1}{4} \rightarrow l_{1+\epsilon} = \frac{v_c}{4f_c}$  (1)

$l_{2+\epsilon} = \frac{\lambda_2}{4} \rightarrow l_{2+\epsilon} = \frac{v_H}{4f_H}$  (2)

and using  $v \propto \sqrt{T}$

Hence  $\frac{l_{1+\epsilon}}{l_{2+\epsilon}} = \frac{\sqrt{T_c} f_H}{\sqrt{T_H} f_c}$   
 $= \sqrt{\frac{293}{303}} \frac{f_H}{f_c}$

Now, using the data  $0.52 + \epsilon = \frac{343}{4 \cdot f_c} = \frac{343}{4 \times 156}$

gives  $\epsilon = 3.0 \text{ cm} (2.97 \text{ cm})$

Now using (1) and (2)

$$\frac{0.52 + 0.03}{l_2 + 0.03} = \frac{\sqrt{293}}{\sqrt{303}} \times \frac{151.2}{156}$$

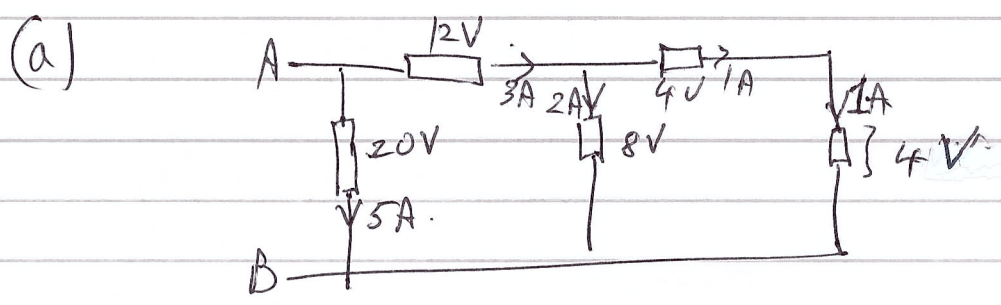
so that  $l_2 = 0.49 \text{ m}$   
 $= 49 \text{ cm}$

[One mark for 52.3cm]

If the best frequency is used to give  $160.8 \text{ Hz}$  in the shorter tube  $l_2 = 52.3 \text{ cm}$ , which is longer. The temperature difference means that you have a lower frequency in a shorter tube.

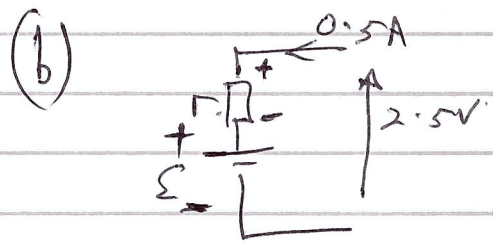
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Question 3 Electrical Circuits

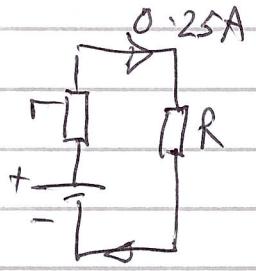


$I_{AB} = 8A$   
 $V_{AB} = 20V$  ✓  
 $R_{AB} = 2.5 \Omega$  ✓

2



$E + 0.5r = 2.5$  ✓



$\frac{E}{0.25} = r + 7.6$

$E = \frac{r}{4} + 1.9$

So  $2.5 - 0.5r = 0.25r + 1.9$

$0.6 = 0.75r$

$r = 0.8 \Omega$  ✓

$E = 2.1 V$  ✓

3

(c) (i)  $\frac{1}{R_{eq}} = \frac{1}{R_A} + \frac{1}{R_B} \Rightarrow R_{eq} = \frac{R_A R_B}{R_A + R_B}$

No mark! We need this result

(ii) i. fraction through  $R_B$  is  $(I) \frac{R_A}{R_A + R_B}$  ✓

ii.  $I_B = \frac{E}{R + \frac{R_A R_B}{R_A + R_B}} \times \frac{R_A}{R_A + R_B} = \frac{E R_A}{R R_A + R_A R_B + R R_B}$  ✓

(iii)  $I_1 = \frac{\mathcal{E}_1}{R}$      $I_2 = \frac{\mathcal{E}_2}{R}$  ✓

11

(iv)  $I_3' = \frac{\mathcal{E}_1}{R_{eq}} \times \text{portion of current through } R_3 \text{ in the parallel section}$   
 $= \frac{\mathcal{E}_1}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} \times \frac{R_2}{R_2 + R_3}$   
 $= \frac{\mathcal{E}_1 R_2}{R_1 R_2 + R_1 R_3 + R_2 R_3}$     N.B.  $\mathcal{E}_1 R_2$  ✓

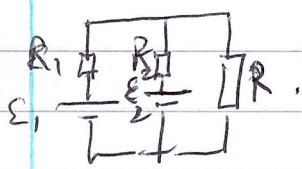
(v) Similarly  $I_3'' = \frac{\mathcal{E}_2 R_1}{R_1 R_2 + R_1 R_3 + R_2 R_3}$  ✓

(vi) Linearity means  $I_3 = I_3' + I_3''$  } need a combined result similar to this ✓

$$= \frac{\mathcal{E}_1 R_2 + \mathcal{E}_2 R_1}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

3

(vii) Divide by  $(R_1 R_2)$   $I_3 = \frac{\frac{\mathcal{E}_1}{R_1} + \frac{\mathcal{E}_2}{R_2}}{1 + \frac{R_3}{R_2} + \frac{R_3}{R_1}}$



The potential across  $R_3$  is  $I_3 R_3 = V$   
 $I_3 R_3 = V = \frac{\frac{\mathcal{E}_1}{R_1} + \frac{\mathcal{E}_2}{R_2}}{\frac{1}{R_3} + \frac{1}{R_2} + \frac{1}{R_1}}$     need  $I_3 R_3$  calculated ✓

(multiply numerator by  $R_3 \equiv$  dividing denominator by  $R_3$ )  
 Just stating this does not get the mark. require  $(I_3 R_3)$  to use it. As  $R_3 \rightarrow \infty$ , the pair of cells are connected to an open circuit (no current in the load resistor) and that is the emf.

$V \rightarrow \text{EMF as } R_3 \rightarrow \infty = \frac{\frac{\mathcal{E}_1}{R_1} + \frac{\mathcal{E}_2}{R_2}}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \quad (0)$

and, as  $R_3 \rightarrow 0$  (short circuit)

$$I_3 \rightarrow \frac{E_1}{R_1} + \frac{E_2}{R_2}$$

and the current in a circuit with internal resistance,  $I_3 = \frac{E}{R+r}$

$$\text{So } \frac{1}{r} = \frac{1}{R_1} + \frac{1}{R_2}$$

already to zero

$$r = \frac{R_1 R_2}{R_1 + R_2} \quad (0)$$

These two results are given. Mark for  $R_3 \rightarrow 0$  to show that this gives  $r$ .

(viii) Max Power when

$$R_3 = \frac{R_1 R_2}{R_1 + R_2}$$

$$= \frac{10 \times 15}{10 + 15} = \underline{\underline{6 \Omega}} \quad \checkmark$$

$$\text{emf} = \frac{50}{\frac{1}{10} + \frac{1}{15}} + \frac{60}{\frac{1}{15}} = \underline{\underline{54 V}} \quad \checkmark$$

ii. Max Power in  $I^2 R_3$

$$= \left( \frac{E}{2R_3} \right)^2 R_3$$

$$= \frac{27^2}{6} = 121.5$$

$$= \underline{\underline{120 W}} \quad \checkmark$$

(Total power dissipated in circuit is 243W)

Charging cells: if  $I_2 = 0$ ,  $I_1 = I_3$ ,  $E_1 = I_1 (R_1 + R_3)$

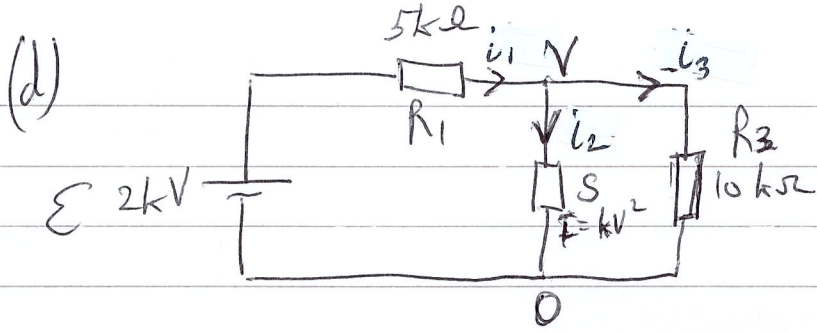
$$E_2 = I R_3 \text{ with } I_2 = 0$$

$$\text{Then } E_1 = \frac{E_2}{R_3} (R_1 + R_2) = E_2 \frac{R_1}{R_3} + E_2 \rightarrow 50 = 60 \frac{10}{R_3} + 60$$

$$\text{or if } E_2 \text{ charges } E_1 \text{ unit, } E_2 = E_1 \frac{R_2}{R_3} + E_1 \quad R_3 \text{ is negative } \times$$

$$60 = 50 \frac{15}{R_3} + 50$$

$$R_3 = \underline{\underline{75 \Omega}} \quad \checkmark \quad \boxed{16}$$



✓ currents specified in diagram.

Kirchhoff:  $\mathcal{E} = i_1 R_1 + V$

} a use of Kirchhoff ✓

and  $i_1 = i_2 + i_3$

given  $i_2 = kV^2$

then  $\mathcal{E} = i_2 R_1 + i_3 R_1 + i_3 R_3$

$= kV^2 R_1 + i_3 (R_1 + R_3)$

$= k(i_3^2 R_3^2) R_1 + i_3 (R_1 + R_3)$

items of  $i_3$  ✓

Substituting  $2000 = 10^{-7} i_3^2 \cdot 10^8 \cdot 5 \times 10^3 + i_3 \cdot 15 \times 10^3$

i.e.  $2 = i_3^2 \times 10 \times 5 + i_3 \times 15$

quadratic in  $i_3$  ✓

$50 i_3^2 + 15 i_3 - 2 = 0$

$i_3 = \frac{-15 \pm \sqrt{225 + 4 \times 50 \times 2}}{100}$

$= \frac{-15 \pm 25}{100}$

$i_3 = \underline{\underline{0.1 A}}$

✓✓

(ii) current doubles to  $i_3 = 0.2 A$

Hence  $V = 0.2 \times 10^4 = \underline{\underline{2000 V}}$

✓

Then  $i_2 = 10^{-7} \times (2000)^2 = 0.4 A$

But  $i_1 = i_2 + i_3$

$= 0.4 + 0.2 = \underline{\underline{0.6 A}}$

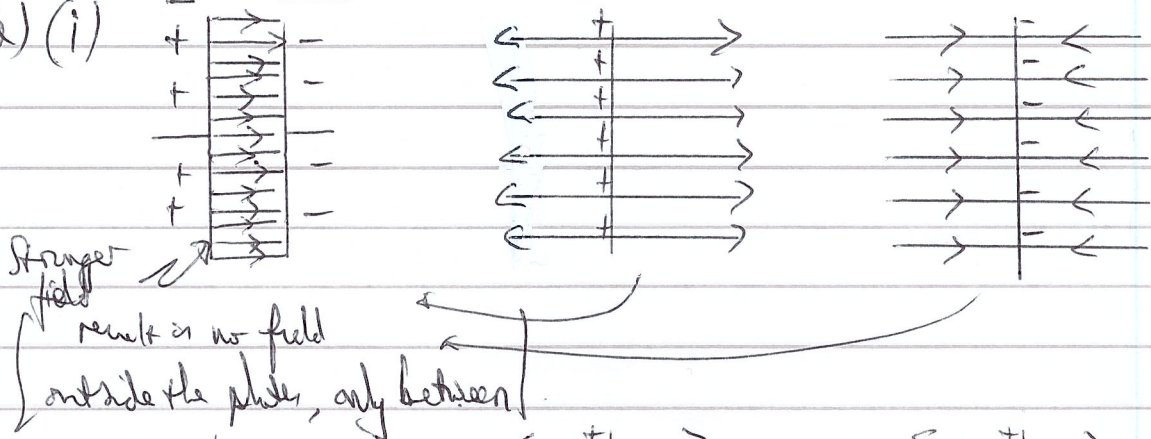
potential across  $R_1$  is  $5 \times 10^3 \times 0.6 = 3 \times 10^3 V$

So  $\mathcal{E} = 3000 + 2000 V = \underline{\underline{5 kV}}$

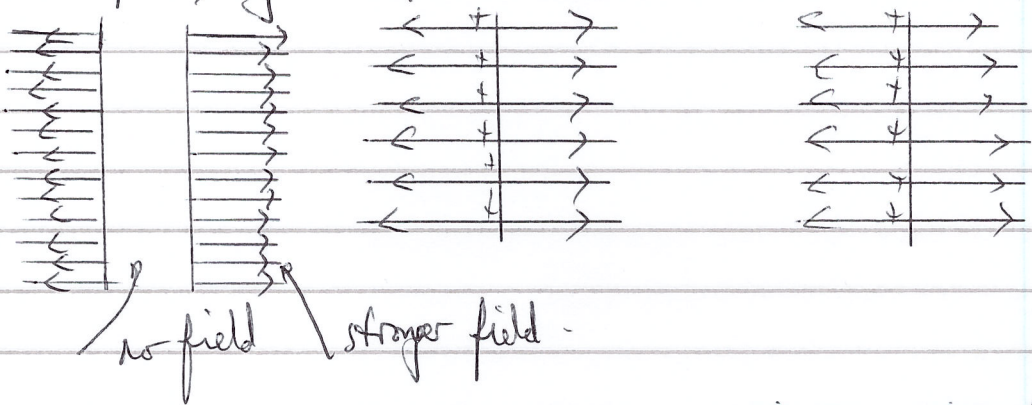
✓  
8

Qn 4. Electric Fields

(a) (i)



(ii)



(iii)

W.D. in separating the plates is  $F \cdot d$ .  
 (constant field or constant charge)  
 So the force between the plates can be obtained from the energy stored

$$F \cdot d = \frac{Q^2}{2C}$$

but  $C = \frac{\epsilon_0 A}{d} \therefore Fd = \frac{Q^2 d}{2\epsilon_0 A}$

$$F = \frac{Q^2}{2\epsilon_0 A}$$

(iv)

In the uniform electric field  $E$  from one plate, the other plate contains charge  $Q$ .

So  $F = Q \cdot E$

Hence  $\frac{Q^2}{2\epsilon_0 A} = Q E$

field,  $E = \frac{1}{2\epsilon_0} \cdot \frac{Q}{A} = \frac{\sigma}{2\epsilon_0}$

(v) Using  $F = \frac{Q^2}{2\epsilon_0 A}$

$$\frac{Q_1^2}{A_1} = \frac{Q_2^2}{A_2}$$

$$\frac{A_2}{A_1} = 2 = \left(\frac{Q_2}{Q_1}\right)^2 \text{ (or other method)}$$

∴ factor of  $\sqrt{2}$  increase.



(vi) equating forces,  $Mg = \frac{Q^2}{2\epsilon_0 A}$



$$(\rho A \Delta h) g = \frac{\sigma^2 A^2}{2\epsilon_0 A}$$

$$\text{or } \rho \Delta h \cdot g \times 2\epsilon_0 = \sigma^2$$

$$10^3 \times 1.5 \times 10^{-3} \times 9.8 \times 2 \times 8.85 \times 10^{-12} = \sigma^2$$

$$\sigma = 16 \mu\text{C}/\text{m}^2$$



or by energy.  $m g \Delta h = (\rho A \Delta h) g \Delta h = \frac{1}{2} \epsilon_0 E^2 \times A \cdot \Delta h$

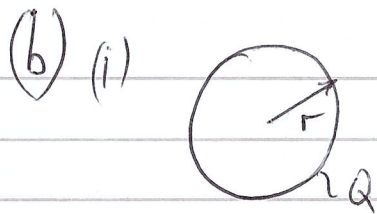
$$\text{energy}/\text{m}^3 = \frac{1}{2} \epsilon_0 E^2$$

$$\rho A (\Delta h)^2 g \times 2\epsilon_0 = \sigma^2 \cdot A \Delta h$$

$$\sigma^2 = \rho \Delta h \cdot g \times 2\epsilon_0 = 16 \mu\text{C}/\text{m}^2$$







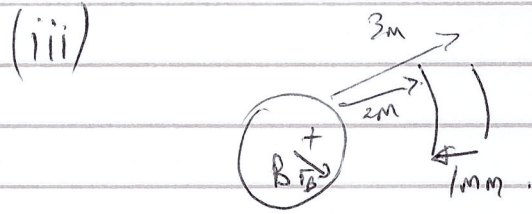
$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r}$$

$$Q = 4\pi\epsilon_0 rV$$

(ii)  $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2}$  for a radial (spherical) field.

or  $E = \frac{V}{r}$

$$E_A \Gamma_A = E_B \Gamma_B$$



$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r} = \frac{kQ}{r}$$

$$k = V(1\text{mm}) \times 1\text{mm} = 2 \text{ kVmm}$$

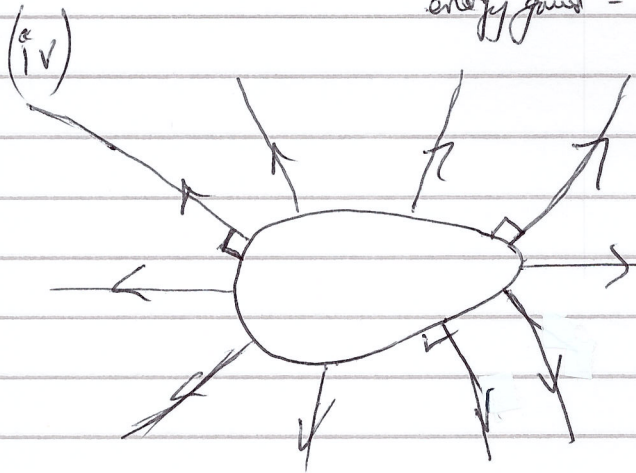
$$V(2\text{mm}) = \frac{2 \text{ kVmm}}{2\text{mm}} = 1 \text{ kV}$$

$$V(3\text{mm}) = \frac{2}{3} \text{ kV}$$

$$\text{energy gained} = e\Delta V = \left(1 - \frac{2}{3}\right) \text{ kV} \times 1.6 \times 10^{-19} \text{ C}$$

$$= 0.53 \times 10^{-18} \text{ J}$$

$$= 0.33 \text{ keV}$$



lines are perpendicular to the surface at the surface  
arrows

(v)  $W = Fd = eE \cdot d$

or  $2.5 \text{ eV} = e \times E \times 7 \times 10^{-6}$

$$E = \frac{2.5}{7 \times 10^{-6}} = 3.6 \times 10^5 \frac{\text{N}}{\text{C}}$$

(vi) Since  $E = \frac{V}{r}$  for a fixed  $V$   $E \propto \frac{1}{r}$ .

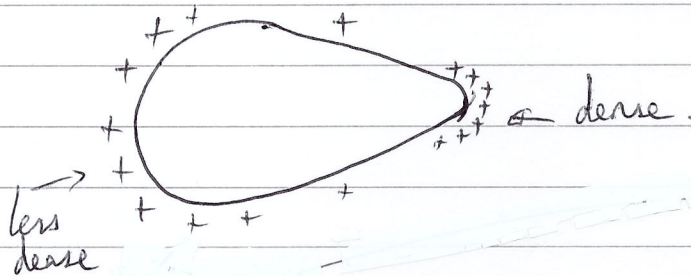
so the smaller (radius) end will have the higher field strength and will break down first.  
Correct end ✓

$$E(5.0 \text{ cm}) = \frac{V_{\text{breakdown}}}{5 \times 10^{-2} \text{ (m)}}$$

$$3.0 \times 10^6 = \frac{V_{\text{breakdown}}}{5 \times 10^{-2}}$$

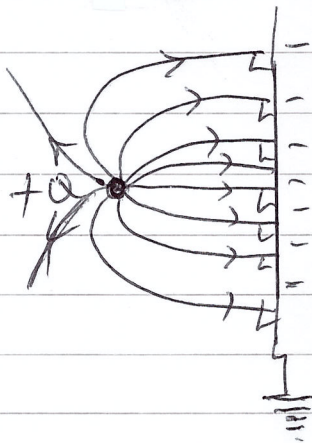
$$V_{\text{breakdown}} = 17860 \text{ V} \\ = \underline{18 \text{ kV}}$$

(vii)



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(c) (i)



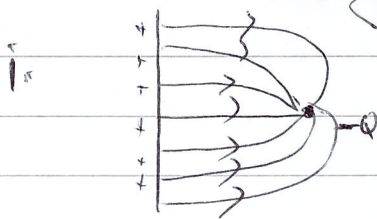
field lines  $\perp$  to plate ✓  
radial close to charge ✓

(ii) negative induced charges attract +Q ✓

ii. Force on plate is to the left ✓

iii. and symmetric about a line through Q. ✓

(iii)



ii. pattern is like charges of  $4.0 \mu\text{C}$  separated by  $4.0 \text{ cm}$ , as the lines are perpendicular to the plate, so no other component of force.

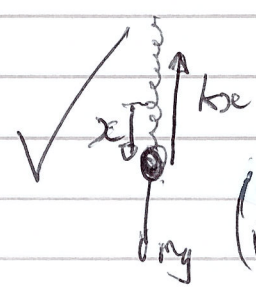
$$F = \frac{1}{4\pi\epsilon_0} \frac{(4.0 \times 10^{-6})^2}{(0.4)^2} = \underline{0.9 \text{ N}}$$

7

Qu 5. Springs + Forces

(a) (i)

Diagram



In equilibrium  $k(l-l_0) = mg$

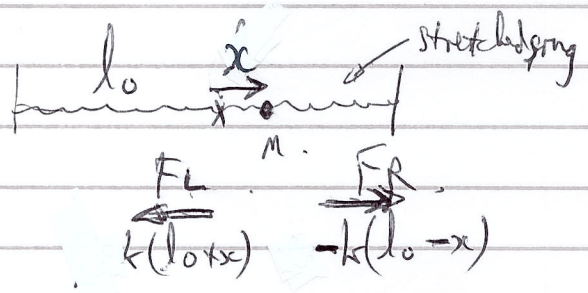
(ii) When displaced downwards  $-k(l+x-l_0) + mg = ma$

Hence  $-Mg + kx + mg = ma$   
 so,  $a = -\frac{k}{m} \cdot x$

Compare with  $a = -\omega^2 x$

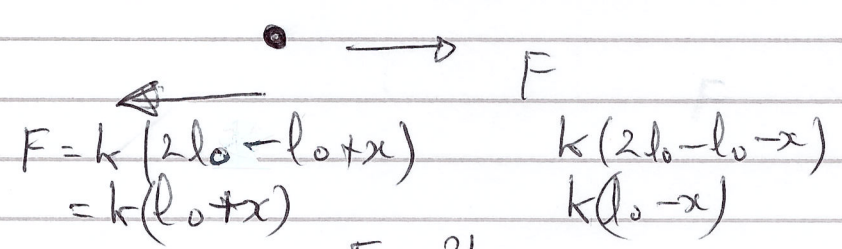
and  $T = 2\pi \sqrt{\frac{m}{k}}$  (no mark for result) 2

(ii)



$= 2kx \implies T = 2\pi \sqrt{\frac{m}{2k}}$

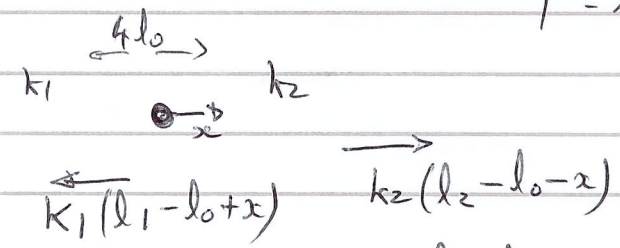
(iii)



$F = 2kx$

$T = 2\pi \sqrt{\frac{m}{2k}}$

(iv)

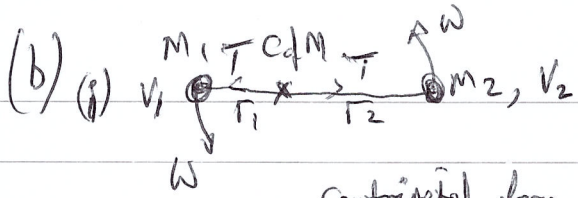


At equilibrium  $k_1(l_1 - l_0) = k_2(l_2 - l_0)$

so,  $F = k_1x + k_2x$

and  $T = 2\pi \sqrt{\frac{m}{k_1 + k_2}}$

2



Centripetal force :  $m_1 r_1 \omega^2 = m_2 r_2 \omega^2 = T$  ✓

$m_1 r_1 = m_2 r_2$

and  $r = r_1 + r_2$

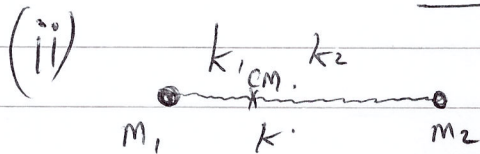
So  $m_1 r_1 = m_2 (r - r_1)$

$m_1 = m_2 \left( \frac{r}{r_1} - 1 \right)$

$\frac{m_1 + m_2}{m_2} = \frac{r}{r_1}$

$r_1 = \frac{r m_2}{m_1 + m_2}$

$T = \frac{m_1 m_2 r \omega^2}{m_1 + m_2}$  ✓



- The CM remain fixed. ✓
- The spring can be considered as two parts  $k_1, k_2$  about the CM.
- The extension  $x$  is the sum of the individual extensions  $x_1, x_2$   
 $x = x_1 + x_2$ .
- The force of extension  $F$  is given by  $F = k_1 x_1$   
 $F = k_2 x_2$   
 $F = k x$ .

So  $\frac{F}{k} = \frac{F}{k_1} + \frac{F}{k_2} \Rightarrow \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$

Since  $T = 2\pi \sqrt{\frac{m_1}{k_1}}$  or  $T = 2\pi \sqrt{\frac{m_2}{k_2}}$   $\frac{1}{k} = \frac{T^2}{4\pi^2 m_1} + \frac{T^2}{4\pi^2 m_2}$   
 $= \frac{T^2}{4\pi^2} \left( \frac{1}{m_1} + \frac{1}{m_2} \right)$

$T = 2\pi \sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}}$  ✓

(c) (i)

$\Delta t / \text{ms}$	$\Delta V / \text{ms}^{-1}$	speed / m/s	av. speed / m/s	distance / m
0-10	30	340 → 310	325	3.25
10-20	50	310 → 260	285	2.85
20-90	245	260 → 15	138	9.6
90-100	15	15 → 0	7.5	0.08
		340		15.8 m

→ from 340 m/s forest.

16 m or 14-18 m 5-marks (with evidence working)

Working - 3x changes of velocities ✓  
 2x average velocities ✓  
 2x distances ± / m of values. ✓  
 Table layout or ordered calculation which can be read ✓

(d) Momentum conservation  $M_{\text{bullet}} \times V_{\text{bullet}} = (M_b + M_{\text{wood}}) V_2$

$$10^{-2} \times 320 = 4 \times V_2$$

$$V_2 = 0.8 \text{ m/s} \quad \checkmark$$

Energy of both (and bullet) goes into elastic potential energy.

Assume linearity i.e.  $F \propto \text{displacement}$ .

which means  $F = kx$

so the energy is given by  $\frac{1}{2} kx^2 \quad \checkmark$

$$\text{So } \frac{1}{2} MV^2 = \frac{1}{2} kx^2$$

$$x^2 = \frac{MV^2}{k}$$

$$= \frac{4 \times 0.8^2}{k}$$

And  $200 = k \times 5 \times 10^{-3}$

$$k = 4 \times 10^4 \text{ N/m} \quad \checkmark$$

$$x^2 = \frac{4 \times 0.8^2}{4 \times 10^4}$$

$$x = 8 \text{ mm} \quad \checkmark$$

4

If we solve the quadratic,

$$\frac{mv^2}{k} + 2\frac{mg}{k}(\Delta h) = (\Delta h)^2$$

$$\Delta h = \frac{2mg}{k} \pm \sqrt{\frac{4m^2g^2}{k^2} + \frac{4mv^2}{k}}$$

$$= \frac{mg}{k} \pm \frac{mg}{k} \sqrt{1 + \frac{mv^2 \cdot k^2}{k \cdot m^2g^2}}$$

$$= \frac{mg}{k} \left( 1 + \sqrt{1 + \frac{v^2 k}{mg^2}} \right)$$

$$= 2.21 \text{ cm} \approx \underline{\underline{2.2 \text{ cm}}}$$

Still ~~ward~~ / ~~mist~~  
if gpe left out

(Any value around 2cm will do -)

(iii)

1.2 cm deflection is not small for the change in gpe.  
Using only gpe  $\frac{1}{2}mv^2 = mgy \Delta h$   
 $v^2 = 2g \Delta h$   
 $v = 0.485 \text{ m/s}$

Using the elastic p.e. stored only  
 $\frac{1}{2}mv^2 = \frac{1}{2}k(\Delta h)^2$   
 $v^2 = \frac{k}{m}(\Delta h)^2$   
 $v = 0.716 \text{ m/s}$

Using  $\frac{1}{2}mv^2 + mg \Delta h = \frac{1}{2}k(\Delta h)^2$

$$v^2 = \frac{k}{m}(\Delta h)^2 + 2g \Delta h$$

$$v = 0.865 \text{ m/s}$$

(this is the RMS of the two individual speeds above)

Using eqn ①, momentum conservation.

$$u = v_1 + v_2$$

$$\sqrt{2gh} = v_1 + 10 \times 0.865$$

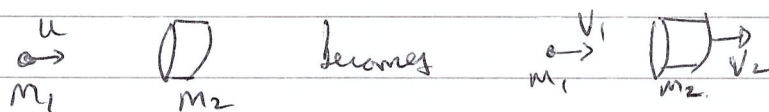
$$v_1 = (-) 2.386 \text{ m/s}$$

$$= \sqrt{2gh}$$

$$\underline{\underline{h = 29 \text{ cm}}}$$

✓ [ 15 ]

(e) General result for an elastic collision between a moving mass  $m_1$ , and an initially stationary mass,  $m_2$ .



Momentum cons.  $m_1 u = m_1 v_1 + m_2 v_2$  ✓

So  $u = v_1 + \frac{m_2}{m_1} v_2$  ①  $\frac{m_2}{m_1} = r$

KE conserved  $\frac{1}{2} m_1 u^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

So  $u^2 = v_1^2 + r v_2^2$  ②

We can rearrange ① as  $u - v_1 = r v_2$

and ② as  $(u - v_1)(u + v_1) = r v_2^2$

If  $u \neq v_1$ , then dividing,

$u + v_1 = v_2$  ③

Add ① and ③

$2u = (1+r)v_2$  ④

Subtracting

$2v_1 = (1-r)v_2$  ⑤

Dividing ④ and ⑤

$\frac{v_1}{u} = \frac{(1-r)}{(1+r)}$  ⑥

Speed of Cylinder

(i) Using ④  $2u = (1+r)v_2 \Rightarrow v_2 = \frac{2u}{(1+r)} = \frac{2\sqrt{2gh}}{1+0.45} = \frac{2\sqrt{2 \cdot 9.81 \cdot h}}{1.45} = 1.14 \text{ m/s}$  (2sf) ✓

(ii) Using energy KE of cylinder + loss of grav pe in lowering = elastic pe gained

$\frac{1}{2} m v^2 + mg \Delta h = \frac{1}{2} k (\Delta h)^2$

This is a quadratic which can be solved for  $\Delta h$ .

To estimate  $\Delta h$ , consider only the elastic pe and KE terms

$\frac{1}{2} m v^2 = \frac{1}{2} k (\Delta h)^2 \Rightarrow (\Delta h)^2 = \frac{m v^2}{k} = \frac{0.45 \times 1.14^2}{1600}$

$\Delta h = 0.019 \text{ m} = 1.9 \text{ cm}$

With this value,  $mg \Delta h = 0.45 \times 9.81 \times 0.019 = 0.084 \text{ J}$

but  $\frac{1}{2} m v^2 = 0.292 \text{ J}$ . So GPE is significant (30%)