BAAO

British Astronomy and Astrophysics Olympiad

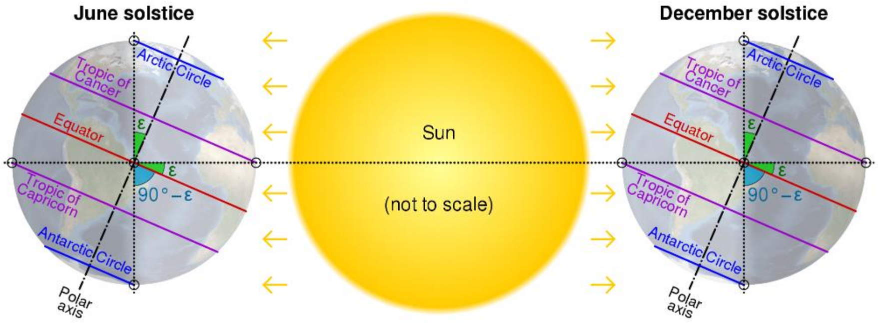
Astronomy & Astrophysics Challenge

September - December 2021

Solutions and marking guidelines

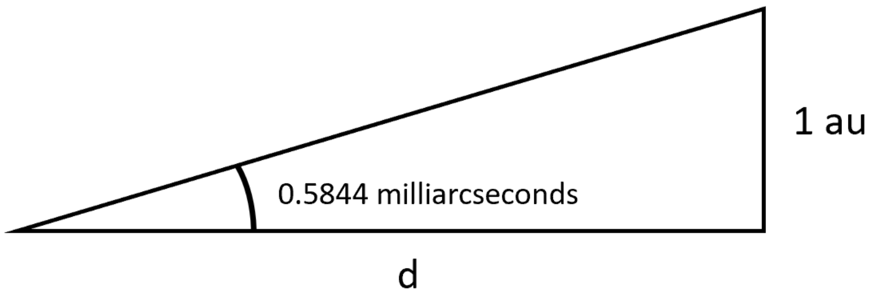
- The total mark for each question is in **bold** on the right-hand side of the table. The breakdown of the mark is below it.
- There is an explanation for each correct answer for the multiple-choice questions. However, the students are only required to write the letter corresponding to the right answer.
- In Section C, students should attempt **either** Qu 13 **or** Qu 14. If both are attempted, consider the question with the higher mark.
- Answers to two or three significant figures are generally acceptable. The solution may give more than that, especially for intermediate stages, to make the calculation clear.
- There are multiple ways to solve some of the questions; please accept all good solutions that arrive at the correct answer. Students getting the answer in a $\boxed{\text{box}}$ will get all the marks available for that calculation / part of the question (students may not explicitly calculate the intermediate stages, and should not be penalised for this so long as their argument is clear)

Question	Answer	Mark
Section A		**10**
1.	C Although many astronauts have used SpaceX to travel into space (and to the ISS), Elon Musk has not (yet) used it himself. Internationally, the Kármán line (at about 100 km) is treated as the edge of space, however in the US the definition is about 80 km, so Richard Branson did go into space by that definition. Dennis Tito was the very first billionaire to go into space in 2001 and visited the ISS for about 8 days.	1
2.	D We can work out the number of squares needed by considering the surface area of the Earth and dividing it by 9 m^2 $$\text{number of squares} = \frac{4\pi(6.37\times10^6)^2}{9} = 5.67\times10^{13}$$ This must be the same as the number of co-ordinates, so the number of words needed must be approximately the cube root: $$n = \sqrt[3]{5.67\times10^{13}} \approx 38\ 400$$ (Note: to ensure all words in the co-ordinate are different and avoid pairings where two of the words are homophones, the actual number is closer to 40,000 – there are about 170 000 words in the English language)	1

3.	C Assuming everything else about the light gathering efficiency of the system stays the same, the new limiting magnitude will simply be a function of the extra photon collecting area (converted onto the magnitude system), so $$m_{new} = m_{old} + 2.5\log\left(\frac{D_{new}^2}{D_{old}^2}\right) = 31 + 2.5\log\left(\frac{6.5^2}{2.4^2}\right) = 33.2$$ Note that the larger telescope means it can gather more photons in the same amount of time, so it can see fainter objects, which corresponds to a larger positive magnitude. (In reality, the JWST has a more efficient optical system, so its expected limiting magnitude is predicted to be closer to 34)	1
4.	C Due to the Earth's axial tilt (ε), there is a zone of the Earth' surface where the Sun can be directly overhead (and hence generate no shadows for vertical sticks) at local midday. The northernmost edge of this zone is the Tropic of Cancer (where it occurs on the June solstice) whilst the southernmost edge is the Tropic of Capricorn (where it occurs on the December solstice, which was the date given in the question).  For latitudes between these it happens twice a year symmetrically around the solstice – for the Equator it happens on the equinoxes. The tilt also means above the Artic Circle and below the Antarctic Circle you get a continuous 6 months of day followed by 6 months of night.	1
5.	D If Mars is in opposition, that means the Sun is on the opposite side of the sky. The Sun is in Taurus in May/June, so 6 months later is when Mars will be in opposition (i.e. November/December). The actual opposition will occur on 8th December 2022.	1
6.	A Centring the list of zodiacal constellations around Aquarius gives: Leo – Virgo – Libra – Scorpio – Sagittarius – Capricorn – **Aquarius** – Pisces – Aries – Taurus – Gemini – Cancer The closest available option to Aquarius is Capricorn.	1
7.	C The right ascension (RA) of the Sun is 0h 00m at the vernal equinox in March. October is 7 months later, so the expected RA is roughly $\frac{7}{12} \times 24^{\mathrm{h}} = 14\ \mathrm{h}$. Spica is the closest to that, whilst also having small declination (so is close to the ecliptic)	1
8.	C Taking the average wavelength of sunlight to be 550 nm, we can work out the energy of each photon and so work out the number of photons received by the eye each second.	1

	received power = received intensity × collecting area $$\therefore P = \frac{L_\odot}{4\pi(1\text{ au})^2} \times \pi r^2 = \frac{3.85 \times 10^{26}}{4\pi(1.50 \times 10^{11})^2} \times \pi(2 \times 10^{-3})^2 = 17\text{ mW}$$ $$\therefore n = \frac{P}{E_{photon}} = \frac{P}{\frac{hc}{\lambda}} = \frac{17 \times 10^{-3}}{\frac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{550 \times 10^{-9}}} = 4.7 \times 10^{16}$$ (The Sun is not a monochromatic source, but the answer does not change greatly when considering the whole spectrum. When close to the horizon there can be strong atmospheric effects that dim the light from the Sun and so the number of received photons per second can be significantly reduced)	
9.	B The formula on page 2 becomes $a^3 = T^2$ when in au and years in the Solar System, and so $$a = \sqrt[3]{T^2} = \sqrt[3]{172^2} = 30.93\text{ au}$$ The aphelion distance would be $$r_{aph} = a(1+e) = 30.93(1+0.94) = 60\text{ au}$$ ∴ the comet is currently at aphelion, and so will be travelling fastest when at perihelion, which is half a period later ∴ $0.5T = 86$ years	1
10.	A Comparing the relative angular velocities of the Earth and the asteroid gives the asteroid's net angular velocity relative to the Earth $$\omega_{rel} = \omega_E - \omega_{ast}$$ $$\therefore \frac{2\pi}{T_{rel}} = \frac{2\pi}{T_E} - \frac{2\pi}{T_{ast}}$$ $$\therefore \frac{1}{3.5} = \frac{1}{1} - \frac{1}{T_{ast}} \therefore T_{ast} = 1.4\text{ years}$$ Using $a^3 = T^2$ we get $a = \sqrt[3]{1.4^2} = 1.25$ au	1
Section B		**10**
11.	a) Reading the mean apparent magnitude off the figure, treating it as a near vertical line and a roughly straight, diagonal line: $$\langle m \rangle \approx 7.1$$ (allow ± 0.1) Correcting for absorption by interstellar dust (known as extinction) will make it brighter, and so the magnitude will become smaller: $$\langle m \rangle_{corr} = \langle m \rangle - 1.42 = 7.1 - 1.42 = 5.68$$ Using the given formula for Cepheids to work out the absolute magnitude: $$\langle \mathcal{M} \rangle = -2.43(\log P - 1) - 4.05 = -2.43(\log 41.5 - 1) - 4.05 = -5.55$$ Using the second given formula for distance: $$d = 10^{0.2(\langle m \rangle_{corr} - \langle \mathcal{M} \rangle + 5)} = 10^{0.2(5.68-(-5.55)+5)} = \boxed{1760\text{ pc}}$$	**[3]** 0.5 0.5 1 1

<table>
<tr><td></td><td>[Allow ecf for adding rather than subtracting the extinction, leading to 6520 pc - this scores 2.5 marks. Watch out for students doing log(41.5 -1) rather than log(41.5) – 1, and allow ecf as appropriate]</td><td></td></tr>
<tr><td></td><td>b)

Based upon the description of parallax in the question:


$d = \dfrac{1\ \mathrm{au}}{\tan\left(\dfrac{0.5844 \times 10^{-3}}{3600}\right)} = 5.29 \times 10^{19}\ \mathrm{m} = \boxed{1710\ \mathrm{pc}}$

[The first mark is for showing an understanding of the situation with a suitable diagram OR for correctly converting the angle to either degrees or radians. Accept small angle approximation if it is converted into radians]

[Note: Because of how a parsec is defined, they may use the formula

$d\ (\text{in pc}) = \dfrac{1}{\theta\ (\text{in arcseconds})} = \dfrac{1}{0.5844 \times 10^{-3}} = 1710\ \text{pc}$

This approach receives full marks]

This gives a reassuringly similar distance to the Cepheid variable method; that relationship was calibrated using distances to nearby stars determined from parallax</td><td>[2]

1

1</td></tr>
<tr><td>12.</td><td>a)

In the Gregorian system the average number of days in a year:
365 (baseline)
+ 1/4 (leap year every 4 years)
– 1/100 (not a leap year if a centurial year)
+ 1/400 (but one centurial year per cycle is a leap year)
$= 365\frac{97}{400} = \boxed{365.2425}$

This is closest to a $\boxed{\text{tropical year}}$
(∴ the seasons occur in the same calendar months every year)

[The second mark can only be awarded if there is some working, and allow ecf for it]</td><td>[2]

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	b) Difference between the two types of year: $$365.256363 - 365.242190 = 0.014173 \text{ days} = \boxed{20 \text{ min } 25 \text{ s}}$$ [Must be in minutes and seconds for the mark. Accept ± 1 sec]	**[1]** 1
	c) We are looking for the number of sidereal years that need to pass (N) such that the number of tropical years that have passed in the same time period is exactly one more (i.e. N + 1) such that the starting points realign $$N \times 365.256363 = (N+1) \times 365.242190$$ $$\therefore N = \frac{365.242190}{0.014173} = \boxed{25770 \text{ (sidereal) years}}$$ [In tropical years this is 25771 years – allow full marks for this so long as it is clear the student understood what they were doing]	**[2]** 1 1
Section C		**10**
13.	a) We need to find the difference between the lunar and solar years: $$365.25 - (12 \times 29.53) = 365.25 - 354.36 = 10.89 \approx \boxed{11 \text{ days}}$$ [Must be to the nearest day for the mark. Accept 10 days too]	**[1]** 1
	b) To find the Gregorian year when the Islamic calendar started, we need to subtract 1429 lunar years converted into Gregorian years from 2008: $$2008 - 1429 \times \frac{354.36}{365.35} = 621.61 \approx \boxed{622 \text{ CE}}$$ [The CE is not required for the mark. Accept 621 too] Using a similar approach to Q12, we can look for the number of Gregorian years that need to pass for the gap between the year in CE and AH to reduce by 1: $$N \times 365.25 = (N+1) \times 354.36 \therefore N = \frac{354.36}{10.89} = 32.54 \text{ years}$$ This means the time to wait is: $$(2008 - 1429) \times 32.54 = 18840.6 \text{ years}$$	**[3]** 1 1 0.5

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Consequently, the final date is:

$$2008 + 18840.6 = 20848.6 \approx \boxed{20849}$$

[Accept 20848 too. If they are working in lunar years, $N = 33.54$ years, so the wait time from 1429 is 19419.6 years, leading to the same final answer]

The real Gregorian date corresponding to the start of the Islamic calendar is Friday 16th July 622 CE, so our calculation has performed well. Here we have suggested that 20849 CE = 20849 AH; in reality, the same year will only be reached in 20874. The reason for the discrepancy is that we do not have enough precision on the numbers used to confidently extrapolate that many millennia into the future, as well as implicitly assuming that the start of the year 2008 CE and 1429 AH coincided (in reality 1429 AH started on 11th Jan 2008 CE) but we are still within 30 years of the answer.
</td><td>0.5</td></tr>
<tr><td></td><td>
c)

The system suggests an average month length of 29.5 days, so the discrepancy per lunar year is

$$12 \times (29.53 - 29.5) = 0.36 \text{ days}$$

Therefore, the number of lunar years that need to pass to generate a leap day are

$$\frac{1}{0.36} = 2.78 \approx \boxed{3 \text{ lunar years}}$$
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[1]

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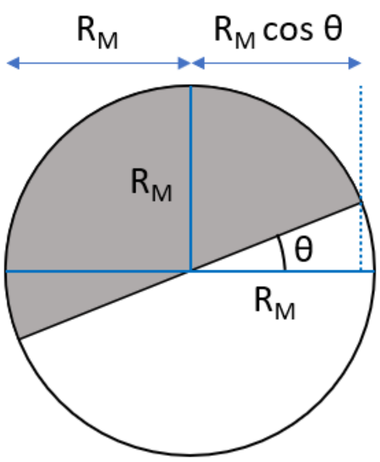
d)

Suitable diagram of the situation
(Here it is viewed from above the lunar North pole)



The shadow fraction is the ratio of the area in shadow over the whole area of the lunar disc, however since the vertical diameter is the same for both, it is only the ratio of horizontal diameters that matters

$$\text{Shadow fraction} = \frac{R_M + R_M \cos\theta}{2R_M} = \frac{1}{2}(1 + \cos\theta)$$

If 0.6% is illuminated, 99.4% is in shadow

$$\therefore 0.994 = \frac{1}{2}(1 + \cos\theta) \therefore \cos\theta = 0.988 \therefore \theta = 0.155 \text{ rad} \ \ (= 8.89°)$$
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[5]

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	Assuming it has a circular orbit then the angular velocity will be constant: $$\therefore \theta = \omega t = \frac{2\pi t}{T}$$ where θ is in radians, T is the lunar period and t is the time since the last new moon $$\therefore t = \frac{\theta T}{2\pi} = \frac{0.155 \times 29.53}{2\pi} = 0.729 \text{ days} = \boxed{17.5 \text{ hours}}$$ [Allow formula for θ for fourth mark in degrees. Must be in hours for the final mark. Some students might work out the shadow fraction by having the ratio of (semicircle + half ellipse) / circle, giving $\frac{\frac{1}{2}\pi R_M^2 + \frac{1}{2}\pi R_M \cdot R_M \cos\theta}{\pi R_M^2}$ which cancels down to the same expression – note that the area of an ellipse was given on page 2 of the paper] This is a sizeable amount of time after the astronomical new moon, meaning that the start date of months in the Islamic calendar in countries that use the sighting method can often only be narrowed down in advance to one of two days, with the final decision of which one it will be only taken on the day.	1 1
14.	a) We are told that the image is 530 arcseconds by 530 arcseconds – printed full size on A4 it should be 15.9 cm by 15.9 cm, so the scale is 3.33 arcseconds per mm The major axis (2a) is approximately 105 mm (allow ± 5 mm) The minor axis (2b) is approximately 75 mm (allow ± 5 mm) $$\therefore a = \frac{1}{2} \times 105 \times \frac{3.33}{3600} \times \frac{2\pi}{360} = 8.5 \times 10^{-4} \text{ rad}$$ and $$\therefore b = \frac{1}{2} \times 75 \times \frac{3.33}{3600} \times \frac{2\pi}{360} = 6.1 \times 10^{-4} \text{ rad}$$ $$\therefore \Omega = \pi ab = \pi \times (8.5 \times 6.1) \times 10^{-8} = 1.6 \times 10^{-6} \text{ steradians}$$ [Allow appropriate credit if their paper has been printed at a different scale, propagating tolerances. Allow full credit for any reasonable estimation method that gives 1.43×10^{-6} sr $< \Omega < 1.81 \times 10^{-6}$ sr] $$I_\nu = B_\nu \Omega \quad and \quad B_\nu = \frac{2\nu^2 k_B T}{c^2} \quad \therefore T = \frac{I_\nu c^2}{2\nu^2 k_B \Omega}$$ $$\therefore T = \frac{7.43 \times 10^{-24} \times (3.00 \times 10^8)^2}{2 \times (3.0 \times 10^9)^2 \times 1.38 \times 10^{-23} \times 1.6 \times 10^{-6}} = \boxed{1670 \text{ K}}$$ [The allowed values of Ω give a range of 1490 K to 1880 K for full credit]	**[6]** 1 0.5 0.5 0.5 0.5 1 1 1

	This shows the nebula is still very warm, as one would expect for the gas to still be ionized, however there is considerable variation between the temperature of different features in the Crab Nebula (e.g. knots, filaments, voids etc.) that our simple model ignores	
	b) Given the distance to the nebula we can work out the physical size of the major axis: $s = d\tan\theta = 2.0 \times 10^3 \times \tan(8.5 \times 10^{-4}) = 1.70\ \mathrm{pc}\ (= 5.2 \times 10^{24}\ \mathrm{m})$ [Allow use of small angle approximation, otherwise ensure calculator is in the right mode e.g. in radians] Assuming constant deceleration, we can use SUVAT to work out the time the nebula has been expanding: $s = vt - \frac{1}{2}at^2$ $\therefore 5.24 \times 10^{16} = (1500 \times 10^3)t - \frac{1}{2}(-15 \times 10^{-6})t^2$ $\therefore t = 3.03 \times 10^{10}\ \mathrm{s} = 962\ \mathrm{years}$ (We have ignored the negative solution, $t = -2.30 \times 10^{10}$ s) Given the photo is from 2017, the year of the supernova is: $2017 - 962 = \boxed{1055}$ [Allow full credit for students using their own correctly propagated value of the angular semi-major axis, as well as full ecf for any outside the allowed range. The expected range of t is from 922 years to 1003 years, leading to the supernova being between 1014 to 1095] The real supernova was observed by Chinese astronomers on 4th July 1054 and remained visible until about 6th April 1056. Since this is one of the few nearby supernova remnants for which we have a clear idea of age, and given its brightness in both radio and X-rays, it is one of the most studied objects outside of the Solar System.	**[4]** 1 1 1 1