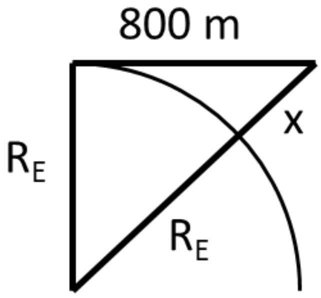
Astronomy & Astrophysics Challenge

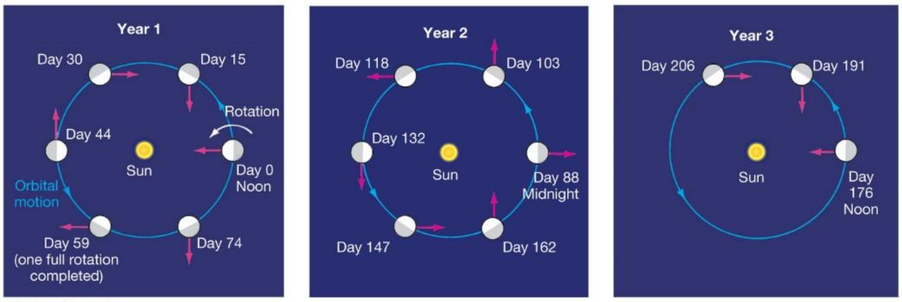
September - December 2019

Solutions and marking guidelines

- The total mark for each question is in **bold** on the right-hand side of the table. The breakdown of the mark is below it.
- There is an explanation for each correct answer for the multiple-choice questions. However, the students are only required to write the letter corresponding to the right answer.
- In Section C, students should attempt **either** Qu 13 **or** Qu 14. If both are attempted, consider the question with the higher mark.
- Answers to two or three significant figures are generally acceptable. The solution may give more to make the calculation clear.
- There are multiple ways to solve some of the questions; please accept all good solutions that arrive at the correct answer.

Question	Answer	Mark
Section A		**10**
1.	C It's an image of the supermassive black hole at the centre of the galaxy M87 and was significant as it is the first time a black hole has been directly imaged. It was done using radio wavelengths and with a telescope with an effective diameter equal to the diameter of the Earth!	1
2.	B In July 1969 was the Apollo 11 mission, which famously landed in the Mare Tranquillitatis ('Sea of Tranquillity').	1
3.	A X-rays are absorbed by the Earth's atmosphere, so an X-ray telescope must be in space (or at least above much of the atmosphere).	1
4.	A This question is easiest solved by recognising which constellations each of the four stars belong to. Aldebaran is in Taurus, which is the 'bull' being hunted by Orion (the 'hunter'), which contains Rigel. Helping Orion are his two 'dogs', with Procyon in Canis Minor and Sirius in Canis Major. As viewed from the UK, Orion is facing the bull and being followed by his dogs, so Aldebaran is the first to rise. A more precise calculation can be done by calculating the length of time that the star is above the horizon with the hour angle, but this is deemed too complicated for this paper.	1

5.	A By taking the photo during a solar eclipse they could see the stars of the Hyades which were behind the Sun at the time. They found that they had been shifted by the gravitational lensing of the Sun, and that the size of the shift was consistent with General Relativity (which predicted a shift twice as large as expected from Newtonian mechanics).	1
6.	B A full moon in Capricorn tells us that the Sun must be on the opposite side of the sky, corresponding to Cancer or Leo. The UK season that corresponds to this is the summer.	1
7.	C Using the given formula: $$g \propto \frac{M}{R^2} \;\therefore\; M \propto gR^2 \quad \therefore \frac{M_{Titan}}{M_{Moon}} = \frac{g_T R_T^2}{g_M R_M^2} = 1.82$$	1
8.	A  Using Pythagoras' Theorem: $$R_E^2 + 800^2 = (R_E + x)^2 \quad \therefore x^2 + 2R_E x - 800^2 = 0$$ Solving this quadratic gives $x \approx 0.05 \text{ m} = 5 \text{ cm}$ (the other root is clearly not applicable to this situation and corresponds to an equivalent triangle out the other side of the planet).	1
9.	D Using the formula given on page 2: $$a = \sqrt[3]{\frac{GM_E T^2}{4\pi^2}} = \sqrt[3]{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times (24 \times 60 \times 60)^2}{4\pi^2}} = 4.22 \times 10^7 \text{ m}$$ So the circumference $= 2\pi a = 2.65 \times 10^8$ m and hence the average separation is that divided by 450, which is 590 km. Note that this is technically the length of the arc between the two satellites, but since the angle between each is less than a degree it means that calculating the direct line of sight distance is only negligibly shorter.	1
10.	D Since $b \propto 1/r^2$ we can say that at Ultima Thule the Sun is 43.4^2 times fainter in brightness than it is at the Earth. Using the formula given on page 2, this corresponds to a change in magnitude of: $$\Delta m = \frac{1}{0.4} \log 43.4^2 = 8.19$$ Finally, we **add** it to apparent magnitude as viewed from the Earth (since it is less bright), so -26.74+8.19 = -18.55. This is much brighter than the full moon hence New Horizons will only risk taking a photo of the Earth once it has finished all of its science, as if the Sun is too close to the field of view it would be bright enough to permanently damage the camera.	1

Section B		10
11.	a) The solar day on Mercury is NOT the same as the sidereal (= relative to the stars) day, which would be easily calculated as 2/3 × 88 days = 59 days [Any student giving that answer for this part receives 0 marks as the question explicitly tells you that it's longer than a Mercurian year]  As shown by the diagram above, Mercury's solar day is 176 Earth days (or two Mercurian years, or an equivalent statement) [One mark can be gained for an attempt at a suitable diagram. The correct answer gains both marks, whether or not there is a diagram] [*Alternative example written answer for 2 marks*: In 88 days Mercury orbits once, and in this time it has rotated 1½ times. Starting from facing the Sun at noon, it would have to rotate once just to keep facing the Sun in its orbit, so no day would have passed by. But we are told it rotates an extra half a rotation (so half a day has passed in its year), hence a whole day takes two Mercurian years.]	2 1 1
	b) For the Sun to appear stationary, the astronaut must walk at Mercury's rotational speed (as defined by the length of a solar day) $v = \frac{2\pi R_{\text{Mercury}}}{T}$ $= \frac{2\pi \times 2440}{2 \times 88 \times 24}$ $= 3.63\ \text{km h}^{-1}$ This is roughly walking speed! [Allow full ecf in this section for their answer to part a)]	3 1 1 1

12.	a)	**2**
	Recognising that they need to determine the gradient of the graph AND use the solid line rather than the dashed one (see comment in caption)	1
	$$H_0 = 510 \pm 10\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$$	1
	b)	**3**
	In SI units, the unit of H_0 becomes:	
	$$\frac{\mathrm{km\ s^{-1}}}{\mathrm{Mpc}} = \frac{\mathrm{m\ s^{-1}}}{\mathrm{m}} = \mathrm{s^{-1}}$$	
	Hence $n = -1$	1
	Converting the value of H_0 into SI:	
	$$510\ \mathrm{km\ s^{-1}\ Mpc^{-1}} = 510 \times \frac{1000}{10^6 \times 3.09 \times 10^{16}} = 1.65 \times 10^{-17}\ \mathrm{s^{-1}}$$	1
	Putting this into the given formula:	
	$$t = H_0^{-1} = (1.65 \times 10^{-17})^{-1} = 6.06 \times 10^{16}\ \mathrm{s} = 1.92\ \mathrm{Gyr}$$	1
	[Must be in Gyr for the final mark. Allow full ecf from part a)]	
	Since Hubble's distances were quite a long way off the currently accepted values, his age of the Universe is much shorter than what we believe it is today, but it was still a challenge to the prevailing idea at the time that the Universe was eternal (the 'Steady State Theory') and was one of the first pieces of evidence that the Universe might instead have a finite age.	

Section C		10
13.	a) First, we calculate the mass difference between four hydrogen nuclei and one helium nucleus: $$\Delta m = 4m_{\mathrm{H}} - m_{\mathrm{He}} = (4 \times 1.674 \times 10^{-27}) - 6.649 \times 10^{-27}$$ $$= 4.7 \times 10^{-29}\ \mathrm{kg}$$ We can then use Einstein's most famous equation ($E = mc^2$) to work out the energy released per reaction $$E_{\mathrm{p-p}} = \Delta mc^2 = 4.7 \times 10^{-29} \times (3.00 \times 10^8)^2 = 4.23 \times 10^{-12}\ \mathrm{J}$$	**1** 0.5 0.5
	b) First, we calculate the number of reactions per second, N: $$N = \frac{L_\odot}{E_{\mathrm{p-p}}} = \frac{3.85 \times 10^{26}}{4.23 \times 10^{-12}} = 9.10 \times 10^{37}$$ Since there are four hydrogen nuclei used per reaction, the total number of hydrogen nuclei fusing per second is 3.64×10^{38} The percentage of hydrogen mass converted into energy is simply the change in mass in the p-p chain divided by the total mass of hydrogen used in the p-p chain: $$\frac{\Delta m}{4m_{\mathrm{H}}} = \frac{4.7 \times 10^{-29}}{4 \times 1.674 \times 10^{-27}} = 0.702\%$$	**2** 0.5 0.5 1
	c) First, we can work out the total mass of hydrogen available for fusion: $$M_{\mathrm{H}} = 13\% \times 71\% \times M_\odot = 1.84 \times 10^{29}\ \mathrm{kg}$$ By using our answer to part b) for the percentage mass change of the hydrogen, we can calculate how much mass will be converted into energy by the process: $$\Delta M_{\mathrm{H}} = 0.702\% \times 1.84 \times 10^{29} = 1.29 \times 10^{27}\ \mathrm{kg}$$ This can be converted into the total energy released by the Sun over its lifetime: $$E_{\mathrm{tot}} = \Delta M_{\mathrm{H}} c^2 = 1.29 \times 10^{27} \times (3.00 \times 10^8)^2 = 1.16 \times 10^{44}\ \mathrm{J}$$ Assuming a constant luminosity, we can then calculate the hydrogen burning lifetime of the Sun: $$t_\odot = \frac{E_{\mathrm{tot}}}{L_\odot} = \frac{1.16 \times 10^{44}}{3.85 \times 10^{26}} = 3.01 \times 10^{17}\ \mathrm{s} = 9.56 \times 10^9\ \mathrm{years}$$ [Must be in years for the final mark]	**3** 0.5 0.5 1 1

	d)	**4**
	First, we can use Kepler's Third Law (from page 2) to calculate the total mass of the binary star system: $$M_{tot} = \frac{4\pi^2}{G}\frac{a^3}{T^2} = \frac{4\pi^2}{6.67 \times 10^{-11}} \frac{(9.1 \times 1.50 \times 10^{11})^3}{(3.83 \times 60 \times 60 \times 24 \times 365)^2}$$ $$= 1.03 \times 10^{32} \text{ kg}$$	1
	We then know the heavier star is 3/4 of the total mass, so must be: $$7.74 \times 10^{31} \text{ kg } (= 38.9\ M_{\odot})$$	1
	Since $E_{\text{tot}} \propto M,\ L \propto M^{3.5}$ and $t = \frac{E_{\text{tot}}}{L}$ we can say that $t \propto M^{-2.5}$	1
	By comparison with the Sun: $$t = t_{\odot}\left(\frac{M}{M_{\odot}}\right)^{-2.5} = 9.56 \times 10^9 \times (38.9)^{-2.5} = 1.01 \times 10^6 \text{ years}$$	1
	[Accept any suitable unit for the final mark]	
	This means that stars that are much heavier than the Sun have considerably shorter main sequence lifetimes – they live fast and die young! Even though the structure and thermal transfers within such a large star will be rather different to the Sun, detailed modelling shows that this simple method gives a reasonable estimate at the correct order of magnitude.	

14.	a) The brightness (i.e. intensity) of the Sun as viewed from Earth is $$b_{\text{Earth}} = \frac{L}{4\pi r^2} = \frac{3.85 \times 10^{26}}{4\pi \times (1.50 \times 10^{11})^2} = 1362 \text{ W m}^{-2}$$ We are told the period of the planet and the stellar mass so we can use Kepler's Third Law to work out its orbital radius and hence (with the stellar luminosity) its brightness: $$a_{\text{d}} = \sqrt[3]{\frac{GM_*T^2}{4\pi^2}}$$ $$= \sqrt[3]{\frac{6.67 \times 10^{-11} \times (0.089 \times 1.99 \times 10^{30}) \times (4.050 \times 24 \times 3600)^2}{4\pi^2}}$$ $$= 3.32 \times 10^9 \text{ m}$$ $$\therefore b_{\text{d}} = \frac{L_*}{4\pi a_{\text{d}}^2} = \frac{5.22 \times 10^{-4} \times 3.85 \times 10^{26}}{4\pi \times (3.32 \times 10^9)^2} = 1450 \text{ W m}^{-2}$$ [This is only about 6% larger than b_{Earth} and so we have verified the brightnesses are comparable.] With the two brightness and the apparent magnitude of the Sun as viewed from Earth we can calculate the different in magnitude and hence the new magnitude: $$\Delta m = 2.5 \log \frac{b_{\text{Earth}}}{b_{\text{d}}} = 2.5 \log \frac{1362}{1450} = -0.068$$ $$m_{\text{new}} = m_{\odot} + \Delta m = -26.74 + (-0.068) = -26.81$$ So TRAPPIST-1 is just as bright as the Sun as viewed from planet d, although it has an angular width almost six times larger (just under 3 degrees) so has a much more imposing presence in the sky. [Accept the modulus of Δm for the fourth mark, and any consistent sign convention for the fifth mark – the student must recognise that since TRAPPIST-1 is brighter, that will correspond to a more negative apparent magnitude]	**5** 1 1 1 1 1

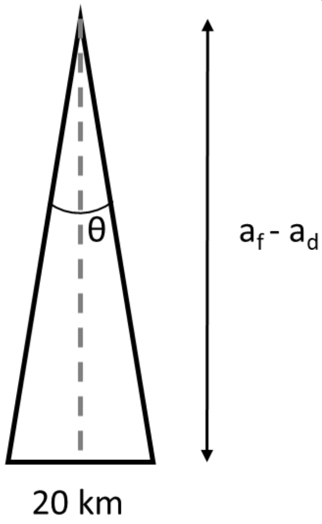
b) **5**

We can use Kepler's Third Law again to work out the orbital radius of planet f:

$$a_\mathrm{f} = \sqrt[3]{\frac{GM_* T^2}{4\pi^2}}$$
$$= \sqrt[3]{\frac{6.67 \times 10^{-11} \times (0.089 \times 1.99 \times 10^{30}) \times (9.206 \times 24 \times 3600)^2}{4\pi^2}}$$
$$= 5.74 \times 10^9 \text{ m}$$

1

We can work out the full angular width of the city, θ, by using its width and distance, and assuming we can use the small angle approximation:



$$\tan\frac{\theta}{2} = \frac{10 \text{ km}}{a_\mathrm{f} - a_\mathrm{d}} \quad \therefore \ \theta \approx \frac{20 \text{ km}}{a_\mathrm{f} - a_\mathrm{d}}$$
$$= 8.26 \times 10^{-6} \text{ rad}$$
$$(= 4.73 \times 10^{-4} \text{ deg} = 1.70 \text{ arcsec})$$

1

Using the given formula to find out the peak wavelength of the star's spectrum:

$$\lambda_\mathrm{max} = \frac{2.90 \times 10^{-3}}{T} = \frac{2.90 \times 10^{-3}}{2511}$$
$$= 1.15 \times 10^{-6} \text{ m}$$

1

This is in the near infrared, and about twice as long a wavelength as the peak wavelength for the Sun

Finally, we can use the given formula to work out the necessary diameter of the telescope by making θ = θ_{min} and λ = λ_{max}:

$$D = \frac{1.22\lambda_{max}}{\theta} = \frac{1.22 \times 1.15 \times 10^{-6}}{8.26 \times 10^{-6}} = 0.171 \text{ m}$$

1

This is a (very) feasible size of telescope to make

1

[Allow ecf for the final mark if they write something consistent with their value of D]

(So, it is likely that if the planets do have cities and astronomers that know how to make telescopes, they are probably aware of each other. The largest telescopes on Earth are around 10 m in diameter, and a 0.7 m telescope would be capable of resolving the city throughout the whole orbit.)