British Physics Olympiad

BPhO Physics Challenge - Mark Scheme

September/October 2019

Instructions **Give equivalent credit for alternative solutions which are correct physics. Generally allow leeway of ± 1 significant figure.**

This is not the tight marking scheme of a competitive exam paper. It is to allow students to engage in problem solving and develop their physics by working through problems requiring explanations, and developing ideas or models. Mark generously to encourage ideas, determination and the willingness to have a go.

1. a) Zero resultant force **AND** Zero resultant moment ✓

[1]

b) (i) centre of mass rises **AND** GPE rises ✓

(ii) small displacements cause a moment about the remaining point of contact, which is in a direction such as to return object to the undisplaced position ✓
Therefore stable ✓

(iii) small displacements cause a moment about the apex, which tends to cause a greater displacement ✓
Therefore unstable

(iv) By symmetry, centre of mass stays at the same height, so displacement from any position (rolling) when resting on a horizontal surface will not lead to a force/couple towards or away from that position. ✓

[5]

c) (i) Diagram to show two concurrent (forces that pass through a common point), equal and opposite forces. ✓

Figure 1: (a) (b)
(a) No resultant force and no moment. (b) This shows a resultant moment so it is not correct - partly because it is not clear what it is showing; do these two forces act on an extended body to produce a moment (a couple), or are they just adrift? The object on which they act is not shown.

Equal and opposite forces acting on a point object will be in equilibrium. But if they act on an extended object, they can produce a moment unless they are in line (which does not have to pass through the centre of mass - just zero displacement between the parallel lines of action of the forces)

(ii) Diagram (or equivalent wording) to show three concurrent, co-planar equal forces ✓
mutually inclined at 120° ✓

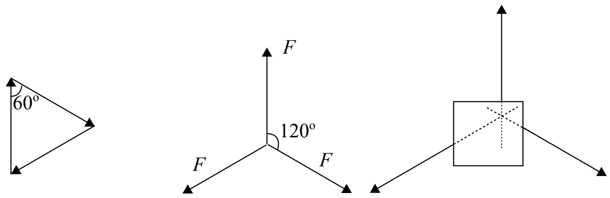

Figure 2: Three equal magnitude forces shown in three diagrams: **left**, the forces add up to zero resultant, but the body is not shown, so we do not know if there is a moment; **centre**, they are concurrent and so have zero moment; **right**, the forces can act on different points of the body, but the concurrent lines of action of the forces produces zero moment.

If the three (equal) forces are drawn tip-to-tail to form a closed triangle, there will be zero resultant force on the body. The three forces must be co-planar for the triangle to close. This is the procedure for adding forces vectorially. To produce zero moment, the three lines of action of the forces (the arrows) must pass through a single point - so this part of the answer requires not a triangle but a point with the equal arrows coming out at angles of $120°$ *(in the same plane). The two forces example is simplistic enough that the two corresponding diagrams are almost equivalent, but not quite: the tip-to-tail diagram corresponds to zero resultant force, and the tail-to-tail corresponds to zero moment.*

(iii) (a) This requires two pairs of concurrent opposite forces. The pairs may be inclined at any angle θ in the plane.

A set of four equal forces arranged as a rhombus (more general than a square) may have zero resultant, but could constitute two couples and so not fulfil the second condition for equilibrium.

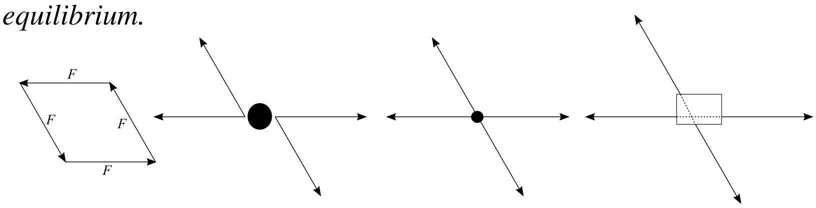

Figure 3: (a) (b) (c) (d)
(a) zero resultant force from rhombus.
(b) these forces can produce a moment on a rigid body.
(c) simple case of coplanar, concurrent forces on a body produces equilibrium.
(d) the forces can act on different points of the body, but the lines of action must be concurrent.

A more general case is when the forces are paired, with each resultant force being concurrent and antiparallel, as shown below in ***Fig 4****.*

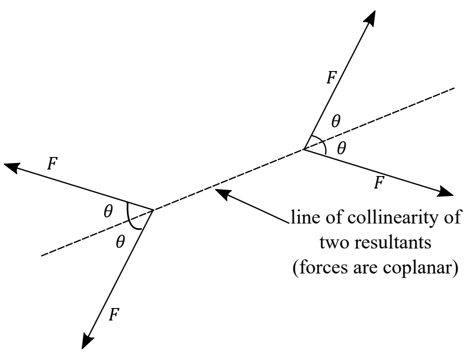

Figure 4

Marking: four forces showing/stated as zero resultant ✓
and zero moment ✓

(b) This question is susceptible to several reasonable answers.
An obvious choice is the four vectors directed from the centroid to the vertices of a regular tetrahedron.
However, other examples fulfilling both conditions will need to be judged on their merits. A more general solution is two pairs of such forces, each pair having the same inclination and therefore the same magnitude for its resultant; the two resultants will need to be collinear and concurrent but not necessarily coplanar, as illustrated below in **Fig 5**.

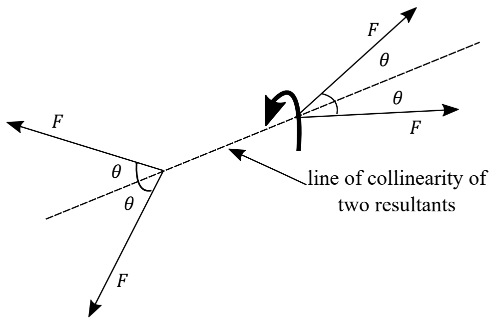

Figure 5

Marking: Any solution providing zero resultant ✓
and zero moment ✓

[7]

13 marks

2. a) (i) excessive current ✓
(ii) overheating, therefore fire hazard ✓

[2]

b) (i)

$$R = \frac{\rho\ell}{A} = \frac{1.7 \times 10^{-8} \times 50 \times 2}{2.5 \times 10^{-6}} \quad ✓$$

$$= 0.68\,\Omega \quad ✓$$

(if $50\,\text{m}$ used as candidate has not allowed for live and neutral conductors in cable, 1 mark lost at this point and then ecf for the rest of the question - given as [])

(ii) lost $V = IR = 20 \times 0.68 = 13.6\,\text{V} \rightarrow V = 230 - 13.6 = 216.4\,\text{V}$ $[223\,\text{V for } 50\,\text{m}]$ ✓

[3]

c) (i) Assume wires are only resistance in the circuit , so $I = V/R = 230/0.68 = 338\,\text{A}$ ✓

(ii) Mass of wires $= V\rho = A\ell\rho = 2.5 \times 10^{-6} \times 100 \times 8940 = 2.24\,\text{kg}$ ✓✓
Rate of heating $= I^2R = 338^2 \times 0.68 = 77.7\,\text{kW}$ (or use $V^2/R, \quad V = 230\,\text{V}$) ✓
Temperature rise in $100\,\text{ms}$ = (rate of heating $\times$ time)$\div mc$ ✓
$= 77.7 \times 10^3 \times 0.1/(2.24 \times 385) = 9.0(4)\,°\text{C}$ $[36.(2)\,°\text{C for } 50\,\text{m}]$ ✓

(iii) Shorter implies less resistance and lower mass; so V^2/R greater AND with less mass, both causing greater temperature rise i.e. $\Delta T \propto 1/\ell^2$. ✓

[7]

12 marks

3. a) (i) $c = \sin^{-1}(1/n) = \sin^{-1}(1/1.52) = 41.1°$ ✓

(ii)

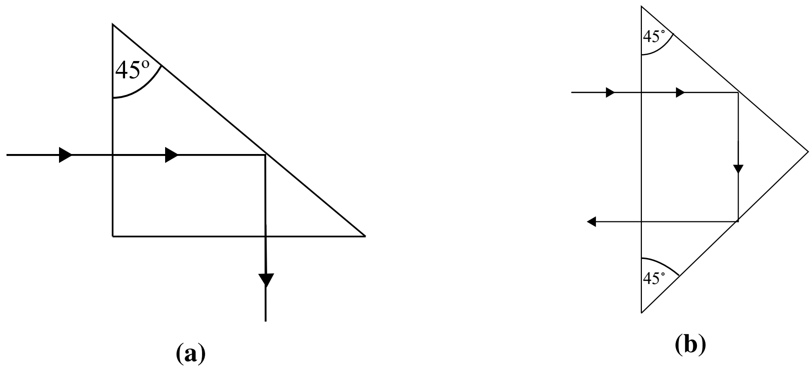

Figure 6

Both diagrams ✓

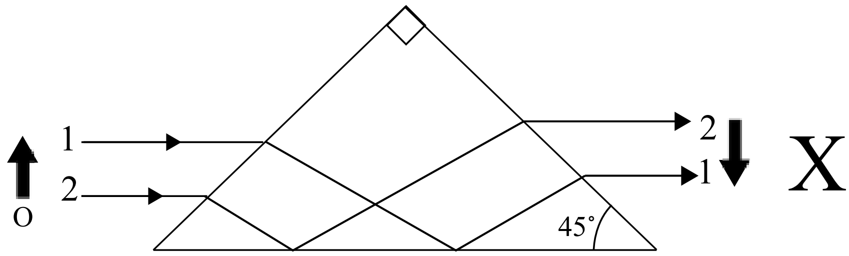

Figure 7

Correct ray paths ✓
Image is inverted ✓
[4]

b) (i) Large diagram completed with angles I, r, i, R, D correctly marked ✓
Diagram must fill more than half the width of the page.

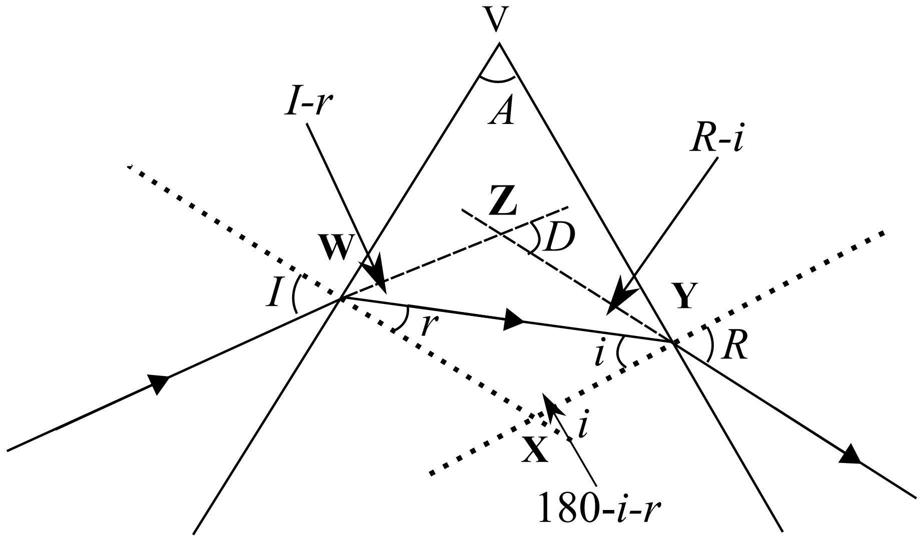

Figure 8

(ii) Show that $i + r = A$
[$\angle$**WXY** $= 180 - \mathrm{i} - \mathrm{r}$ and $\angle$**VWX** $= \angle$**VYX** $= 90°$
internal angles of quadrilateral add to $360° \rightarrow A = 360 - 180 - (180 - i - r) = i + r$] ✓

Show that $D = I + R - A$
[D is external angle of Δ**YWZ** so is equal to the sum of the two opposite interior angles i.e $(I - r) + (R - i) = I - r + R - i = I + R - A$ using result immediately above] ✓

(iii) Show that $I = (A + D)$ when path symmetrical
[In symmetrical case, $R = I$ so from second item in (ii), the result follows] ✓

Show that $r = \frac{1}{2}A$
[In symmetrical case, $r = i$ so from first item in (ii) result follows] ✓

Show that $\sin \frac{(A+D)}{2} / \sin \frac{A}{2} = n$
[Apply Snells law at **W**, using I and r as derived] ✓

It is of interest to note that this symmetrical path corresponds to *minimum deviation*.

(iv) This effect is dispersion.

$$n = \frac{\sin \frac{(60+D)}{2}}{\sin 30}$$

so that $n = 1.62 \rightarrow D_{400} = 48.2°$
$n = 1.59 \rightarrow D_{700} = 45.3°$

Range is 2.9° (or the two values given) ✓
and **BLUE** has the greater angle of deviation. ✓

[8]

12 marks

4. a) (i) condensation **(owtte)** - a phenomenon of "clumping together" ✓

(ii) At constant pressure, the volume of an **ideal** gas would be proportional to the (kelvin) temperature, and decrease to zero volume at absolute zero. ✓

A **real** gas would decrease to a (small) but finite volume. ✓

In the case of both these effects, the gas would condense to a liquid then usually to a solid, which both occupy a finite volume

[3]

b) All five items correct

P is the pressure of the gas of volume V, n is the number of moles,

R is the (molar) gas constant, T is the kelvin or absolute temperature ✓

[1]

c) (pressure units are in kPa) $AP \equiv$ atmospheric pressure

Mouth uppermost, $p = AP + 10$ ✓

mouth downwards, $p = AP - 10$ ✓

Apply Boyle: $p_1V_1 = p_2V_2$, so $(AP + 10) \times 63 = (AP - 10) \times 77$ ✓

Solve to give $AP = 100$ units $= 100\,\mathrm{kPa}$ ✓

[4]

d) (i)
$$n = n_{\mathrm{large}} + n_{\mathrm{small}} = \frac{pV_{\mathrm{large}}}{RT} + \frac{pV_{\mathrm{small}}}{RT}$$ ✓
$$= \frac{1.0 \times 10^5}{(8.31 \times 300)}(1.0 \times 10^{-2} + 1.0 \times 10^{-3})$$
$$= 0.44(1)\,\mathrm{moles}$$ ✓

(ii) Total number of moles remains the same, so ✓

$$0.441 = \frac{p}{R}\left(\frac{V_1}{T_1} + \frac{V_2}{T_2}\right) = \frac{p}{R}\left(\frac{1.0 \times 10^{-2}}{300} + \frac{1.0 \times 10^{-3}}{350}\right)$$ ✓
$$= \frac{p}{R} \times 3.62 \times 10^{-5}$$
$$\text{So} \quad p = \frac{0.441 \times 8.31}{3.62 \times 10^{-5}} = 101\,\mathrm{kPa}$$ ✓

[5]

13 marks

END OF SOLUTIONS