BAAO
British Astronomy and Astrophysics Olympiad

Astronomy & Astrophysics A2 Challenge

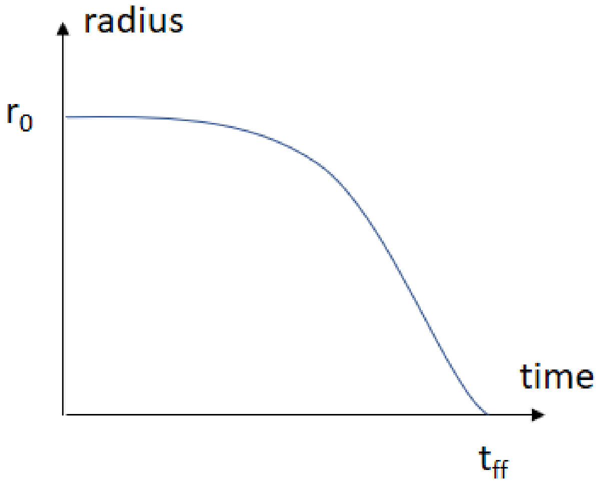
September - December 2017

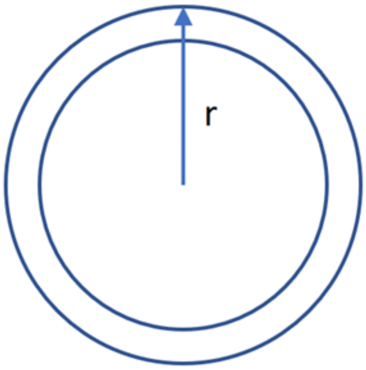
Solutions and marking guidelines

- The total mark for each question is in **bold** on the right-hand side of the table. The breakdown of the mark is below it.
- There is an explanation for each correct answer for the multiple-choice questions. However, the students are only required to write the letter corresponding to the right answer.
- In Section C, students should attempt **either** Qu 13 **or** Qu 14. If both are attempted, consider the question with the higher mark.
- Answers to two or three significant figures are generally acceptable. The solution may give more to make the calculation clear.
- There are multiple ways to solve some of the questions; please accept all good solutions that arrive at the correct answer.

Question	Answer	Mark
Section A		**10**
1.	B Only a collision of black holes could explain the detected gravitational waves, as the objects needed to be about 30 times the mass of the Sun.	1
2.	C The centre of our galaxy is in Sagittarius, given the name Sagittarius A. Very close to the galactic centre is the radio source Sagittarius A*, which is believed to be a supermassive black hole.	1
3.	C The rings of Saturn are surprisingly thin!	1
4.	D Even though it's a dwarf planet, Pluto's surface area of 17.6×10^6 km^2 is slightly bigger than Russia's (17.0×10^6 km^2).	1
5.	C You need the centripetal acceleration to be equal to the value of *g*. Since the centripetal acceleration = $r\omega^2$ $\therefore \omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{9.81}{5 \times 10^6}} = 1.4 \times 10^{-3}\ \mathrm{rad\ s^{-1}}.$ Hence, it will take 4486 s to orbit once (about 1 hour and 15 minutes), so will complete 19.26 rotations a day.	1

6.	D The declination of the Sun varies from +23.5° in the summer solstice to -23.5° in the winter solstice, since that's the angle the Earth's rotational axis is tilted relative to the plane of the solar system. It is 0° at the equinoxes and varies like a sine wave in between these four points over the course of the year.	1
7.	A Only the latitude is relevant here, and so the most northern city will experience the longest day around the summer solstice. If the question had asked for the longest day during December, then it would have been the most southern city instead (Sydney).	1
8.	B For objects in the Solar System, if the semi-major axis is measured in AU and the time is measured in years then Kepler's Third Law can be simplified as $T^2 = a^3$. If a = 4 AU, then $T = \sqrt{(4^3)}$ = 8 years.	1
9.	D If light source B is twice as far away but appears equal in brightness, then it must be 4 times as luminous. When source A is moved to 2r, it's new brightness, $b_{new} = 1/4\ b_{original}$. When source B is moved to 3r its new brightness, $b_{new} = 4/9\ b_{original}$ ∴ ratio = (1/4)/(4/9) = 9/16.	1
10.	B Using the formula from the bottom of page 2 of the paper, a difference of 5 magnitudes corresponds to a factor of 100 in brightness. Since brightness is $\propto 1/r^2$, that means r has increased by a factor of 10.	1
Section B		**10**
11.	a) Angular radius of the Sun: $$\theta = \frac{R_\odot}{1\ \text{AU}} = \frac{6.96 \times 10^8}{1.50 \times 10^{11}} = 4.64 \times 10^{-3}\ \text{rad} = 0.266°$$ Angular speed of the Sun: $$\omega = \frac{180°}{12\ \text{hours}} = 0.25°\ \text{min}^{-1}$$ Extra time: (the factor of 2 is due to there being both a sunrise and sunset) $$t_{\text{angle}} = 2 \times \frac{\theta}{\omega} = 2 \times \frac{0.266}{0.25} = 2.13\ \text{mins}$$	**3** 1 1 1
	b) Extra time due the atmospheric diffraction: $$t_{\text{atmosphere}} = 2 \times \frac{\theta}{\omega} = 2 \times \frac{0.6}{0.25} = 4.8\ \text{mins}$$ Total extra time: $$t_{\text{total}} = 4.8 + 2.13 = 6.93\ \text{mins}$$ So, there is almost 7 more minutes of day than night on an equinox. [Note: allow students that recall (rather than calculate) an angular radius of the Sun of 0.25°, giving a total time of 6.8 mins]	**2** 1 1

12.	a) When r << r_0, then $r/r_0 \approx 0$ so $\cos\theta = 0 \therefore \theta = \pi/2$ Putting this θ into the first equation: $$t_{\mathrm{ff}} = \frac{\pi/2}{\left(\frac{8\pi G\rho_0}{3}\right)^{1/2}} = \sqrt{\frac{3\pi}{32G\rho_0}}$$ (There must have been some attempt to simplify the equation for this mark – though be generous) The time of collapse is independent of the starting radius of the cloud (which is not what you might have first expected)	**3** 1 1 1
	b) $$t_{\mathrm{ff}} = \sqrt{\frac{3\pi}{32G\rho_0}} = \sqrt{\frac{3\pi}{32 \times 6.67 \times 10^{-11} \times 5.0 \times 10^{-16}}} = 2.97 \times 10^{12}\ \mathrm{s}$$ $$= 94\ 000\ \mathrm{years}$$ [Note: must express the answer in years for the mark]	**1**
	c) Graph of radius of the cloud against time:  [Note: allow any graph demonstrating this sort of behaviour (spends most time near r_0 then collapses quite rapidly as t_{ff} is reached) so long as the student has sensibly labelled the graph]	**1**

Section C		10
13.	a)  Surface area of a spherical shell = $4\pi r^2$ Thickness of a very thin shell = $v\Delta t$ So, volume of thin shell = $4\pi r^2 v\Delta t$ $\rho = \frac{M}{V} = \frac{\Delta M}{4\pi r^2 v\Delta t} \therefore \frac{\Delta M}{\Delta t} = 4\pi r^2 v\rho$ [Note: first mark is for calculating volume of a thin shell, and second is for suitable rearrangement]	**2** 1 1
	b) $\frac{\Delta M}{\Delta t} = 4\pi r^2 v\rho$ $= 4\pi \times (1.50 \times 10^{11})^2 \times 500 \times 10^3 \times \left(\frac{7 \times 1.67 \times 10^{-27}}{100^{-3}}\right)$ $= 1.65 \times 10^9 \text{ kg s}^{-1}$ $= 2.62 \times 10^{-14} \text{ M}_\odot \text{ year}^{-1}$ [Note: first mark is for either correct unit conversion of the density **or** for getting the value in kg s^{-1}]	**2** 1 1
	c) $v_{\text{esc}} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{6.96 \times 10^8}}$ $= 6.18 \times 10^5 \text{ m s}^{-1}$ $= 618 \text{ km s}^{-1}$	**1**
	d) $\text{KE density} = \frac{KE}{V} = \frac{\frac{1}{2}\Delta M v^2}{4\pi r^2 v\Delta t} \quad \left(= \frac{1}{2}\rho v^2\right)$ At the boundary $P_{\text{ISM}} = \frac{\Delta M}{\Delta t}\frac{\frac{1}{2}u_\infty^2}{4\pi r^2 u_\infty}$ $\therefore r = \sqrt{\frac{u_\infty}{8\pi P_{\text{ISM}}}\frac{\Delta M}{\Delta t}} = \sqrt{\frac{6.18 \times 10^5}{8\pi \times 10^{-13}} \times 1.65 \times 10^9} = 2.01 \times 10^{13} \text{ m}$ = 134 AU	**4** 1 1 1 1

	[Note: first mark is for any correct attempt at a formula for the kinetic energy density, whilst the second is for realising that at the boundary $v = u_\infty$. If they are using the version of KE density with ρ they must show they realise that the value is different to the one used earlier (i.e. scale with $1/r^2$ or similar) to get the remaining marks. They must have given the distance in AU for the final mark]	
	e) The boundary is not spherical.	**1**
14.	a) $$r = a\frac{m_2}{m_1 + m_2}$$ Substituting in the numbers $$r = 5.20 \times 1.50 \times 10^{11} \times \frac{1.90 \times 10^{27}}{1.99 \times 10^{30} + 1.90 \times 10^{27}} = 7.44 \times 10^8 \text{ m}$$ $$= 1.07 \text{ R}_\odot$$ [Must make the comparison with the solar radius (e.g. give the final answer in those units) to get the final mark]	**2** 1 1
	b) Dividing the numerator and denominator by m_2: $$r = a\frac{1}{\frac{m_1}{m_2} + 1} \therefore \frac{m_1}{m_2} = \frac{a}{r} - 1$$ $$\frac{m_{\text{Pluto}}}{m_{\text{Charon}}} = \frac{19570}{1.83 \times 1187} - 1$$ = 8.01 [Note: first mark is for some attempt to get a formula for the mass ratio. Allow 2 out of 3 marks if the student gets the reciprocal mass ratio (i.e. 1/8 = 0.125)]	**3** 1 1 1
	c) Value of *a* when condition is met $$R_E = \frac{a_{new}}{\frac{M_{\text{Earth}}}{M_{\text{Moon}}} + 1} \therefore a_{new} = 6370 \times (83.1 + 1) = 5.357 \times 10^5 \text{ km}$$ Distance to cover $$d = a_{new} - a = 5.357 \times 10^5 - 3.844 \times 10^5 = 1.513 \times 10^5 \text{ km}$$	**3** 1 1

	So, moving at 4×10^{-5} km year^{-1}, the time needed is $$t = \frac{1.513 \times 10^5}{4 \times 10^{-5}} = 3.8 \times 10^9 \text{ years}$$	1
	d) Overestimate The condition will be met when the Moon is at apogee (furthest away in its orbit) before it meets the condition at the average distance **OR** Underestimate Since the condition will only be met at every point in the Moon's orbit when it satisfies it at perigee (closest in its orbit), which will only happen after it has already met the condition at the average distance [Note: must have a justification – no first mark for a bald statement of 'over/underestimate']	**2** 1 1 1 1