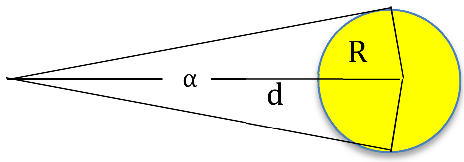
British Olympiad in Astronomy and Astrophysics
April 2015
Solutions and marking guidelines for the BOAA Competition Paper

The total mark for each question is in **bold** on the right hand side of the table. The breakdown of the mark is below it. There are multiple ways to solve some of the questions, so please accept all the good solutions that arrive at the correct answer.

Question	Answer	Mark
Section A		**20**
1.	C	2
2.	B	2
3.	D	2
4.	C	2
5.	A	2
6.	C	2
7.	A	2
8.	C	2
9.	C	2
10.	B	2
Section B		**10**
11.	a. **Answer**: 42,200 km	**3**
	On a circular orbit, the centripetal force is due to the gravitational force:	
	$$F_{cen} = F_{grav}$$	
	$$m\omega^2 r = \frac{GmM_{Earth}}{r^2}$$	1
	$$r = \sqrt[3]{\frac{GM_{Earth}T^2}{4\pi^2}}$$	1
	$T = 24$ hours, therefore the radius of the geostationary orbit is:	
	$$r = 4.22 \times 10^7 \text{ m} \approx 42{,}200 \text{ km}$$	1
	b. **Answer**: 6 hours	**2**
	In the case when the satellite is orbiting the Earth with a period of $T_0 = 24$ hours, but in the opposite direction to Earth's rotation, the relative angular velocity is:	
	$$\omega = 2\omega_0 = \frac{4\pi}{T_0}$$	1
	The observer can be considered as being stationary on Earth and the satellite revolving with ω. Because the radius of the geostationary orbit is larger than the radius of the Earth, the satellite will be visible above the horizon for the observer for half of its (relative) orbit, covering an angle of 180° (or π). The time the satellite is visible for the observer is:	
	$$\Delta t = \frac{\pi}{\omega} = \frac{T_0}{4} = 6 \text{ hours}$$	1
	[Since the radius of the orbit is only 6 R_E, the visibility of the satellite is less than 6 hours. In fact it is 5.42 hours.] This occurs twice in a 24 hour period, so strictly it is 12 hours (or 10.8 h). Either answer of 6 h or 12 h gains the mark.	

12.	a. **Answer**: 630 nm From the classical Doppler effect: $\frac{v}{c} \approx z \approx 0.3$. This is a reasonable approximation as long as $v \ll c$, otherwise the full relativistic Doppler effect has to be used. Doppler shift formula: $$\frac{\lambda - \lambda_0}{\lambda_0} = \frac{v}{c} \approx z$$ Where $\lambda_0 = 486.1$ nm − the rest wavelength. Hence: $$\lambda = \lambda_0(1 + z)$$ The observed (redshifted) wavelength is: $$\lambda = 631.9 \text{ nm } \approx 630 \text{ nm}$$	**3** 1 1 1
	b. **Answer**: 4 billion years The time it takes the light (travelling at speed c) from the galaxy to reach us is: $$\Delta t = \frac{r}{c}$$ According to Hubble's law the distance to the galaxy is: $$r = \frac{v}{H_0}$$ Hence, $$\Delta t \approx \frac{z}{H_0} \approx \frac{0.3}{72 \text{ km s}^{-1}\text{Mpc}^{-1}} \sim 4 \text{ billion years}$$ In the equation above notice that the unit of H_0 is s^{-1} and take care when doing the conversion from Mpc to km.	**2** 1 1
Section C		**20**
13.	a. From Figure 1 identify that the maximum of the eclipse occurs between 09:24 and 09:34, therefore in the image taken at 09:31.	**1**
	b. The brightness of the solar disc is proportional to the visible surface area. The apparent magnitude of the Sun is $m_1 = -26.74$ corresponding to a brightness of F_1. During the eclipse, the Moon covers 90% of the solar disc, thus the visible area of the Sun is only 10%. The brightness of the solar disc during the maximum of the eclipse, is: $$F_2 = 0.1\,F_1$$ This corresponds to an apparent magnitude m_2. Inverting the Pogson's formula, the apparent magnitude of the Sun during the eclipse is: $$m_2 - m_1 = -\frac{\log_{10}\frac{F_2}{F_1}}{\log_{10} 2.512}$$ $$m_2 = m_1 - 2.5\log_{10}\frac{F_2}{F_1}$$ $$m_2 = -26.74 - 2.5\log_{10} 0.1$$ $$m_2 = -24.24$$	**3** 1 1 1

	The Sun was 2.5 magnitudes less bright during the eclipse. It appeared to be slightly dimmer outside, indicating that only 10% of the Sun was able to provide sufficient light to continue our daily activities. Even with a small percentage of the Sun being visible, it is still 40,000 brighter than the full Moon, whose apparent magnitude is -12.74.	
	c. Longyearbyen (latitude $78°13'$ N) is situated above the Arctic circle (latitude $66°34'$ N $= 90° - 23°27'$) and it experiences polar night during the whole month of December (from November to February, more specifically). Hence, the Sun is not visible above the horizon during these months and a solar eclipse would not be observable. Luckily, the total solar eclipse occurred in March.	**1**
	d. Total solar eclipses would no longer be visible from Earth when the angular size of the Moon will be smaller than the angular size of the Sun. To calculate the angular sizes of the Sun and of the Moon, respectively, make use of a diagram such as the one below:  From the diagram, the apparent diameter of the object is: $$\alpha = 2\sin^{-1}\left(\frac{\mathrm{R}}{\mathrm{d}}\right)$$ For small angles, $\sin\alpha \approx \tan\alpha \approx \alpha$ (in radians), so any of these is acceptable. The most favourable case for the total eclipse not to occur is for the Moon to have the largest possible angular size, and the Sun the smallest possible angular size. This happens when the Moon is at perigee (nearest point to the Earth) and the Earth at aphelion (furthest point from the Sun). First calculate the distance to the Sun, r_{max} and its angular diameter. From the diagram of the ellipse on page 1, the distance to the aphelion is: $$r_{max} = a + c = a_E(1 + e_E) = 152.1 \times 10^6 \text{ km}$$ The angular diameter of the Sun (and of the Moon for total eclipses not to occur) is: $$\alpha = 0.524°$$ Now calculate where the Moon must be, d_{min} and hence obtain a_M. Then calculate the change of a_M from current value and, knowing the rate, calculate the time taken. The distance the Moon needs to be situated at is: $$d = \frac{R_M}{\sin\frac{\alpha}{2}} \approx 380{,}000 \text{ km}$$ This will be the perigee of the new orbit, d_{min}. Again, from the diagram of the ellipse on page 1, the distance to the perigee is: $$d_{min} = a - c = a_M(1 - e_M)$$ Hence, the new semi-major axis of the Moon will be: $$a_M = \frac{d_{min}}{1 - e_M} \approx 402{,}120 \text{ km}$$ At a rate of 3.82 cm/year, the Moon will move from the current semi-major axis of 385,000 km to 402,120 km in: $$\Delta t \approx 450 \text{ million years}$$ In 450 million years we will only be able to observe annular solar eclipses from Earth.	**7** 1 1 1 1 1 1 1

14	a. The percentage of the star's disc covered by the planet is 1.65%. The light-curve has high quality data, with an error of only ~ 0.05% [Accept 1.6 $\pm$ 0.1% or 1.7 $\pm$ 0.1% and the errors propagated in the following calculations]	**1** 0.5 0.5
	b. The covered area of the star, during the transit is: $$A_{planet} = \pi R_{planet}^2$$ The percentage determined in a) of $f = 0.0165$ corresponds to: $$f = \frac{A_{planet}}{A_{star}} = \frac{R_{planet}^2}{R_{star}^2}$$ Therefore, the ratio of the radius of the planet and of the star is: $$\frac{R_{planet}}{R_{star}} = \sqrt{f} = 0.128$$	**2** 1 1
	c. Use Newton's third law (which gives the proportionality of Kepler III but with constants): $$\frac{T^2}{a^3} = \frac{4\pi^2}{G(M_{\text{star}} + m_{\text{planet}})}$$ Neglect $m_{\text{planet}} \ll M_{\text{star}}$ and find the radius of the planet's orbit: $$a = \sqrt[3]{\frac{GM_{star}T^2}{4\pi^2}}$$ Using the values in the question, $M_{\text{star}} = 1.15\ M_{\text{solar}}$ and the period of $T = 3.525$ days radius of the planet's orbit is: $$a = 0.047\ \text{AU}$$	**2** 1 1
	d. From the light curve (Figure 2), the total transit time (from the beginning of the drop, to its end) is: $$\Delta t_{total} \approx 0.13\ \text{days}\ \approx 3.12\ \text{hours}$$ The transit time is equivalent to the duration of an eclipse from first to last contact, as seen in Figure 3. During the eclipse, the centre of the planet's disc moves a distance equal to $2(R_{star} + R_{planet})$. Because the star and the planet are practically at the distance from the observer, the speed at which the transit occurs is equal to the circular speed of the planet around the star, v_{planet}: $$v_{planet} = \frac{2(R_{star} + R_{planet})}{\Delta t_{total}} = \frac{2\pi a}{T}$$ Hence, $$R_{star} + R_{planet} = \pi \frac{\Delta t_{total}}{T} a \sim 8.15 \times 10^5\ \text{km}$$ And using the ratio in b), 0.128, the radius of the star and of the planet are: $$R_{star} = 7.23 \times 10^5\ \text{km} \approx 1.04\ R_{Sun}$$ $$R_{planet} = 0.92 \times 10^5\ \text{km}\ \approx 1.32\ R_{Jupiter}$$	**3** 1 1 0.5 0.5