Q1 2015 BPhO Paper 2 Solutions

(a)
(i)

$$E = (R_0 + R)\alpha I_0$$

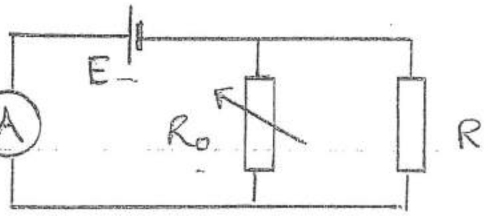
$$\therefore \quad I_0 R_0 = (R_0 + R)\alpha I_0$$

$$R_0 = (R_0 + R)\alpha$$

$$R = \frac{R_0(1-\alpha)}{\alpha}$$ 1

Range of α: $0 \leq \alpha \leq 1$ & $\infty \geq R \geq 0$ 1

(ii)



$$E = \alpha I_0 \left(\frac{1}{R} + \frac{1}{R_0}\right)^{-1}$$

$$I_0 R_0 = \alpha I_0 \left(\frac{R R_0}{R + R_0}\right)$$

$$R = \frac{R_0}{\alpha - 1}$$ 1

Range of α; $1 \leq \alpha \leq \infty$ & $\infty \geq R \geq 0$ 1

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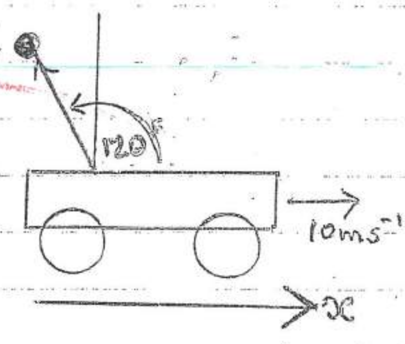
(b) If $(-v_x')$ is the velocity of ball's component in x-direction relative to train, then as the lady observes only vertical motion, relative to her the x-component is zero i.e.



$$0 = -v_x' + 10$$

$v_x' = +10 \text{ ms}^{-1}$ in the backward direction 1

ie ball thrown backwards relative to train at 10 ms^{-1}.

Thus, as train has no vertical component of velocity, the vertical components of the ball's velocity relative to the train, v_y', and that relative to the lady, v_y, are equal

$$v_y = v_y' = |v_x'| \tan 60 = 10\sqrt{3} \text{ ms}^{-1} = 17.3 \text{ ms}^{-1}$$ 2

Q1 2

(b) Thus both observe that the ball reaches a height, h, given by 1

$$v_y^2 = 2gh$$

$$h = \frac{(10\sqrt{3})^2}{2(9.81)} = 15.3\ m$$ 1

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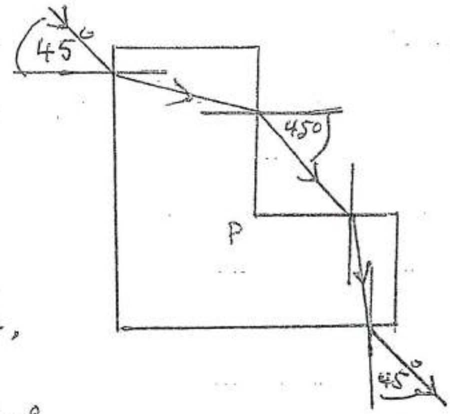
Q1 (c)(i) The beam will emerge from vertical section of glass block parallel to initial beam as the angle of refraction at both surfaces are equal
Similarly for the horizontal section, angle of incidence is 45° and consequently angle of emergence is 45°; parallel to initial beam



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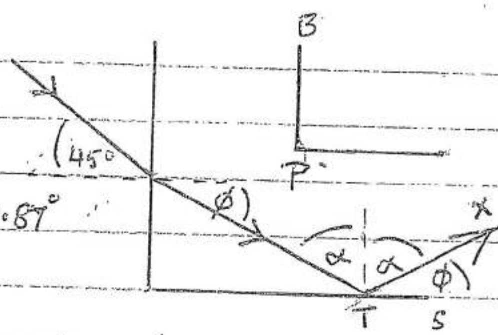
(ii) Using the notation in diagram



$$\frac{\sin 45°}{\sin\phi} = 1.5$$

Giving $\phi = 28.13°$ and $\alpha = 90 - 28.13° = 61.87°$ 1

Critical angle, θ_c, given by

$$\frac{\sin 90°}{\sin\theta_c} = 1.5 \qquad \text{ie } \theta_c = 41.81°$$ 1

So the ray is totally internally reflected - ad infinitum at the two horizontal glass faces 1

As $\hat{STX} = \phi < 45°$ the ray cannot be incident on the vertical face PB 1

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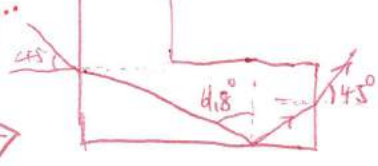
(ii)

* Give ~~full~~ 3 marks if students determine that ray undergoes total internal reflection.
Also give 4 ~~full~~ marks if student shows ray to exit glass at ~~opposite end~~ vereical face eg.



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Q1

(d) Let R_G be the radius of the orbit of Ganymede and M_G its mass. M_J is the mass of Jupiter. Then if ω is the angular velocity of Ganymede,

$$M_G R_G \omega^2 = \frac{G M_G M_J}{R_G^2}$$

$$R_G^3 \omega^2 = G M_J \qquad (1)$$

If the Earth has mass M_E and radius R_E then for mass m on the surface

$$mg = \frac{G m M_E}{R_E^2}$$

$$G M_E = g R_E^2 \qquad (2)$$

From (1) and (2) *

$$\frac{M_J}{M_E} = \frac{R_G^3 \omega^2}{R_E^2 g}$$

$$= \frac{(1.07\times10^9)^3 \left(2\pi/(7.16\times24\times60\times60)\right)^2}{(6.38\times10^6)^2 (9.81)}$$

$$= 316$$

$$\underline{M_J = 316 M_E}$$

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*Alternatively one could substitute the value of M_E

(e)

If δl_c and δl_T are the extensions of the copper and tungsten wire respectively then the Young's modulii are:

$$12.4\times10^{10} = \cancel{\frac{100\times9.81}{\pi(0.50/2)^2\,\delta l_c}} \longrightarrow (1)\ \frac{100\times9.81}{\pi\left(0.5\times10^{-3}/2\right)^2 \delta l_c}$$

$$35.5\times10^{10} = \frac{100\times9.81}{\pi(d/2)^2\,\delta l_T} \qquad (2)$$

Q1 Also $\delta l_c + \delta l_T = 6.00 \times 10^{-2}$ ③

Substituting into ③ from ① and ②

For correct equation

$$6.00 \times 10^{-2} = \frac{100 \times 9.81}{\frac{\pi}{4}} \left[\frac{1}{(0.50 \times 10^{-3})^2 \, 12.4 \times 10^{10}} + \frac{1}{d^2 (35.5 \times 10^{10})} \right]$$

$$\frac{6.00 \times 10^{-2} \left(\frac{\pi}{4}\right)}{10^2 \times 9.81} = \frac{1}{(0.50 \times 10^{-3})^2 \, 12.4 \times 10^{10}} + \frac{1}{d^2 (35.5 \times 10^{10})}$$

$$0.4804 \times 10^{-4} = \frac{4.00 \times 10^{-4}}{12.4} + \frac{1}{d^2 (35.5 \times 10^{10})}$$

$$0.1578 \times 10^{-4} = \frac{1}{d^2} (0.02817 \times 10^{-10})$$

$$d = 0.423 \text{ mm}$$

1 1 1 1

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(f) Activity proportional to number of radioactive nuclei present. Also

$$N = N_0 e^{-\lambda t} \quad ①$$

So

$$\frac{N}{N_0} = \frac{1.2 \times 10^2}{2.0 \times 10^2} = 0.60 \quad ②$$

Now

$$\lambda = \ln 2 / T \quad \text{where } T \text{ is half life}$$
$$= -0.692/T \quad ③$$

Substituting into ① from ② and ③

$$0.60 = \exp(-0.692t/T)$$

or $0.692t = T \ln(1/0.60)$

Sub^g. for T, $= (5.7 \times 10^3)(0.510)$ years

$$t = 4.2 \times 10^3 \text{ years}$$

1 1 1 1 1

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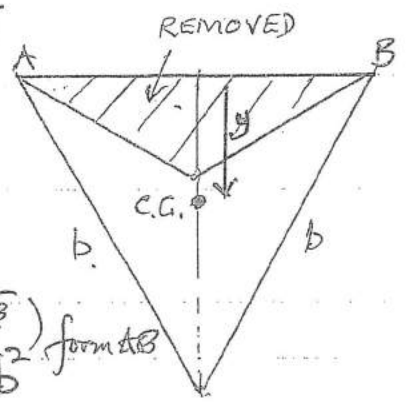
Q1

(g) If one divides the triangle up into thin strips parallel to the side under consideration. The c.g. of each strip is at its mid point. The line joining these c.g.s is the median which terminates at the mid point of the side under consideration. As all triangles formed by a strip and the vertex are similar triangles the median must be a straight line. 1

Area of original triangle $\frac{1}{2}b^2\frac{\sqrt{3}}{2} = \frac{b^2\sqrt{3}}{4}$ 1



Area of removed triangle $\frac{1}{2}b^2\frac{\sqrt{3}}{2}\left(\frac{1}{3}\right)$

$= \frac{1}{4}\frac{\sqrt{3}}{3}b^2$ 1

C.G. of removed triangle $= \frac{1}{3}\left(\frac{1}{3}b\frac{\sqrt{3}}{2}\right)$ from AB

Area of remaining plate $= \frac{1}{6}\sqrt{3}\,b^2$ 1

Taking moments about the upper 'base' of the removed triangle, the c.g. is a distance y from this line along the median given, by subtraction, by

$$\left(\frac{1}{6}\sqrt{3}b^2\right)y = \left(\frac{1}{3}b\frac{\sqrt{3}}{2}\right)\frac{b^2\sqrt{3}}{4} - \frac{1}{3}\left(\frac{1}{3}b\frac{\sqrt{3}}{2}\right)\frac{1}{4}\left(\frac{\sqrt{3}}{3}\right)b^2 \quad 2$$

$$= b^3\left(\frac{1}{8} - \frac{1}{3^2\times 8}\right) = \frac{b^3}{9}$$

$$\underline{y = \frac{2\sqrt{3}}{9}b} \quad 1$$

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Alternative solutions acceptable.

Q 1

(h) The effective 'g' in the lift is $(g+\alpha)$ vertically down 1 1

If U-tube accelerating as in the diagram, the effective vector $\underline{g}$ is given by

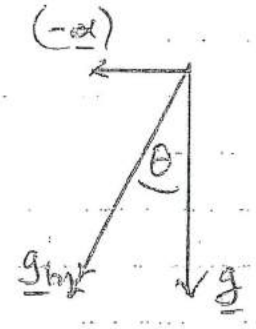
$$\underline{g}_h = \underline{g} - \underline{\alpha}$$

$$g_h = \sqrt{g^2+\alpha^2}$$ 1

at angle θ to vertical given by

$$\theta = \tan^{-1}\left(\frac{\alpha}{g}\right) \quad (1)$$ 1

(θ to left of vertical)



The liquid surface is always perpendicular $\underline{g}_h$ 1

The surfaces in the U-tube are perpendicular to $\underline{g}_h$ (broken line in diagram)
Consequently

$$\tan\theta = \frac{h}{L}$$

$$= \frac{\alpha}{g} \quad \text{from (1)}$$ 1

Thus $\underline{\alpha = \frac{h}{L}\, g}$ 1

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Q1 (i) In the time taken for light to travel from the rotating mirror to the fixed mirror and back again to the rotating mirror, Δt, the rotating mirror must have rotated through half the angle subtended by source and receiver at the rotating mirror, $\Delta\theta$, as the returning beam to the receiver has a rotational speed that is twice that of the mirror. 2

Now

$$\Delta t = 2(0.30\times10^{3})/c$$
$$= \frac{0.60\times10^{3}}{3.00\times10^{8}}\ s$$
$$= \frac{1}{2}\left(\frac{0.60}{0.30\times10^{3}}\right)\frac{1}{\omega}$$ 1

$$\underline{\omega = 0.50\times10^{3}\ \text{rads.s}^{-1}}$$ 1

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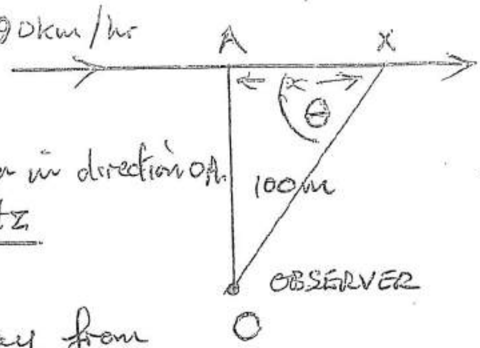
Q1 (j)



(i) No Doppler effect heard by observer when car at A as no motion in direction OA. Frequency heard by observer is 400 Hz

(ii) At X source of sound moving away from observer with a speed of $V = 90\cos\theta$ km/hr

$$= \frac{90 \times 10^3}{60 \times 60} \cos\theta \ ms^{-1}$$

$$V = 25.0 \cos\theta \ ms^{-1}$$

The Doppler shift, frequency ν, with the car frequency of ν_0 is given by

$$\nu = \frac{\nu_0}{1 + \frac{V}{V_s}}$$ (1) from (A) below

where v is the velocity of sound = 343 ms^{-1}.

Thus
$$\nu = \frac{400}{1 + \frac{25.0 \cos\theta}{343}}$$

$$\cos\theta = \frac{x}{\sqrt{x^2 + 10^4}}$$

$$\nu = \frac{400}{1 + \left(\frac{25.0}{343}\right) \frac{x}{\sqrt{10^4 + x^2}}} \ Hz$$

$\rightarrow 0.0729$

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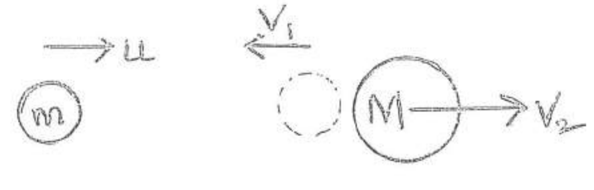
Q1(k)



(i) Conservation of momentum $\qquad \underline{mu = MV_2 - mV_1}$ (1) $\qquad$ 1

Conservation of energy $\qquad \underline{\tfrac{1}{2}mu^2 = \tfrac{1}{2}MV_2^2 + \tfrac{1}{2}mV_1^2}$ (2) $\qquad$ 1

(ii) From (1) $\qquad m(u+V_1) = MV_2 \qquad$ (3)

From (2) $\qquad m(u-V_1)(u+V_1) = MV_2^2 \qquad$ (4) $\qquad$ 3

Dividing (3) into (4) $\qquad \underline{u - V_1 = V_2} \qquad$ (5)

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(l) If second slit moves through an angle θ, or $\theta+2\pi$, or … $\theta + 2n\pi$ (for n integer), particles with velocity v will get through if the time taken Δt to travel from first to second disc satisfies

$$v\Delta t = d$$

But
$$\Delta t = \frac{(\theta + 2\pi n)}{\omega}$$ 2

So
$$v = \frac{d\omega}{(\theta + 2\pi n)} \qquad n = 0, 1, 2, \ldots$$ 1

Substituting numerical values

$$v = \frac{1.0\,(24000)(2\pi)}{\left(\frac{\pi}{3} + 2\pi n\right)}$$ 1

$$v = \frac{1.44 \times 10^{5}}{1 + 6n} \ \text{m min}^{-1} \qquad n = 0, 1, 2, \ldots$$ 1

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* Note that ω is given in rpm so units of v are in m min^{-1}, if student gives v in units of m s^{-1} then answer must be factor 60× smaller.

$$V = \frac{2400}{6n+1} \ \text{m/s} \quad (n = 0, 1, 2, \ldots)$$

Q2

(a) Reversing V, reverses the direction of current in EF. But the 'reversed' network is identical to original network by symmetry so i_{EF} should be in the same direction as in original circuit. Consequently this can only occur if

$$i_{EF} = 0 \quad 1$$

Thus the network consists now of resistances in series and parallel only.

$$R_{AE} + R_{EB} = 2R$$

This is in parallel with $R_{AB} = R$.

Thus total resistance from top face $= \left(\frac{1}{2R} + \frac{1}{R}\right)^{-1} = \frac{2}{3}R$ (1) 1

$R_{CF} + R_{FD} = 2R$ is in parallel with $R_{CD} = R$

Total resistance $\frac{2}{3}R$ (same as (1))

This is in series with R_{AC} and R_{BD} giving

$$2R + \frac{2}{3}R = \frac{8}{3}R \quad (2) \quad 1$$

Now resistances (1) and (2) are in parallel.

Finally resultant resistance, from (1) and (2),

$$R_{AB} = \left(\frac{3}{2R} + \frac{3}{8R}\right)^{-1} \quad 1$$

$$\underline{R_{AB} = \frac{8}{15}R} \quad \frac{1}{5}$$

(b) The 'faces' ABDC and AEFC can be interchanged by symmetry, so

$$i_1 = i_2 \quad 1$$

Also by symmetry currents in BD and EF must be identical, thus requiring

$$i_{BE} = i_{FD} = 0 \quad 1$$

(b) R_{AC} can now be determined by adding resistors in series and parallel.

$R_{ABDC} = 3R$ which is in parallel R in AC

Total resistance $= \left(\frac{1}{3R}+\frac{1}{R}\right)^{-1} = \frac{3}{4}R$ | 1

$R_{AEFC} = 3R$ in parallel with $\frac{3}{4}R$

Final total resistance

$$R_{AC} = \left(\frac{1}{3R} + \frac{4}{3R}\right)^{-1}$$ 1

$$\underline{R_{AC} = \frac{3R}{5}}$$ 1

5

(c) Total Resis. $R_{AD} = \dfrac{V_{AB} + V_{BD}}{(i_1+i_2+i_3)}$ 1

$$= \frac{(i_1+i_3)}{i_1+i_2+i_3} R. \quad \text{(A)}$$ 1

By reversing V, and symmetry,

$$i_{BD} = i_3$$
$$i_{CD} = i_1$$ 1
$$i_{DF} = i_2$$

Using Kirchoff's first rule:

At B, $i_{BE} = (i_1 - i_3)$

At E, $i_{EF} = (i_2 + i_1 - i_3)$ 1

At C, $i_{CF} = (i_3 - i_1)$

Using Kirchoff's second rule round ABE,

$$i_1 + (i_1 - i_3) - i_2 = 0$$

ie $\quad 2i_1 - i_3 - i_2 = 0 \quad (1)$ 1

Round AEFC $\quad i_2 + (i_2 + i_1 - i_3) - (i_3 - i_1) - i_3 = 0$

ie $\quad 2i_1 + 2i_2 - 3i_3 = 0 \quad (2)$ 1

Subtracting (1) from (2), $\quad 2i_3 = 3i_2 \quad$ ie $i_3 = \frac{3}{2} i_2 \quad (3)$ 1

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Substituting $i_3 = \frac{3}{2} i_2$ into ①

$$i_1 = \frac{5}{4} i_2 \qquad ④$$ 1

Substituting ③ and ④ into Ⓐ gives

$$\underline{R_{AD} = \frac{11}{15} R}$$ 2

10

Q3

17

(a) Along the magnetic field the velocity of the mass m is

$$\underline{v\cos\theta.} \qquad (1)$$ 1

This remains constant. 1

Perpendicular to the field the speed is $v\sin\theta$ and the charge is acted on by the force $Bqv\sin\theta$ perpendicular to the velocity. So the motion in this plane is circular with radius R given by 2

$$\frac{mv^2\sin^2\theta}{R} = Bqv\sin\theta$$ 2

$$\therefore \quad \underline{R = \frac{mv\sin\theta}{Bq}} \qquad (2)$$ 1

Consequently the charge has a helical trajectory along the direction of the magnetic field. 1

8

(b) The time for one rotation is $T = \frac{2\pi R}{v\sin\theta}$ 1

From (2) $$T = \frac{2\pi m}{Bq} \quad \text{(independent of } \theta\text{)}$$ 1

Thus the pitch of the helix is, from (1),

$$v\cos\theta T = \underline{\frac{2\pi m v\cos\theta}{Bq}} \qquad (3)$$ 1

3

(c) If θ small, $\cos\theta \approx 1$, 1

from (1) velocity component along field $= v$ for all θ. 1

from (3) $$\text{pitch} = \frac{2\pi mv}{Bq}$$ 1

For small θ all velocities have same pitch. Thus ?

Q3 12

if charges injected into the field at the same point they will all pass through a point P after travelling a distance equal to the pitch, 2

$$L = \frac{2\pi m v}{Bq}$$

If $v = 10^7 \, ms^{-1}$, $B = 10^{-4}\, T$

$$L = \frac{2\pi (9.11\times10^{-31}) 10^7}{10^{-4} (1.60\times10^{-19})}$$ 1

$$L = 3.6 \text{ m}$$ 1

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(d) The only alteration would involve m. m would be relativistic mass given by 1

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$ 1

2

20

Q4

(a) (i)

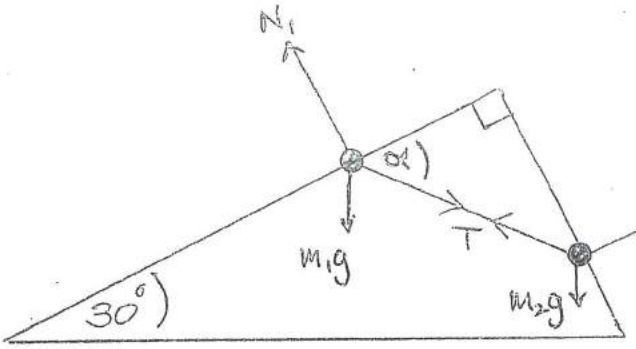
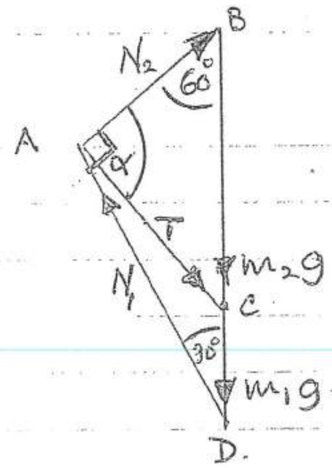


DIAGRAM Correct Forces 4

Introduce normal reactions N_1 and N_2 perpendicular to surfaces for m_1 and m_2 respectively

SOLUTION I Vector Diagram of forces on m_1 and m_2, with common T.



Correct vector diagram/s 2

Using sin rule in triangle ACD

$$\frac{m_1 g}{\sin(90-\alpha)} = \frac{T}{\sin 30^\circ} \quad (1)$$ 1

Using sin rule in triangle ABC

$$\frac{m_2 g}{\sin\alpha} = \frac{T}{\sin 60^\circ} \quad (2)$$ 1

Dividing (1) by (2) and simplifying

$$\frac{\sin\alpha\, m_1}{\cos\alpha\, m_2} = \frac{\sin 60}{\sin 30} = \sqrt{3}$$ 1

$$\tan\alpha = \frac{m_2}{m_1}\sqrt{3} = 3\sqrt{3} \quad \text{as } m_2 = 3m_1$$ 1

16(a)

Q4

Thus $\underline{\alpha = 79 \cdot 11^\circ}$ 1

(ii) (vi) From (2) $T = m_2 g \sin 60 / \sin\alpha$ 1

$= (0{\cdot}300)(9{\cdot}81)\left(\frac{\sqrt{3}}{2}\right) / 0{\cdot}9820$ 1

$\underline{T = 2{\cdot}60 \text{ N}}$ 1

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Q4 (a) (i)

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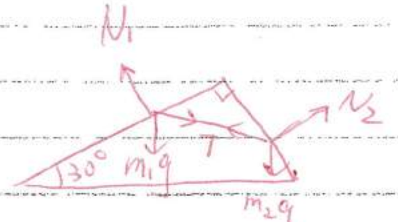
ALTERNATIVE

SOLUTION 2

(~~2~~ 4 MARKS FOR DIAGRAM ~~2~~ 4)



For m_1 resolving along slope,

$$T\cos\alpha = m_1 g\cos 60°$$ 1

$$T\cos\alpha = \tfrac{1}{2} m_1 g \quad \text{as } \cos 60° = \tfrac{1}{2}$$ 1 } 2

$$2T\cos\alpha = m_1 g \quad (1)$$

For m_2, resolving along the slope,

$$T\cos(90° - \alpha) = m_2 g\cos 30°$$ 1

$$T\sin\alpha = \tfrac{\sqrt{3}}{2} m_2 g$$ 1 } 2

$$2T\sin\alpha = \sqrt{3}\, m_2 g \quad (2)$$

From (1) and (2)

$$\tan\alpha = \sqrt{3}\,\frac{m_2}{m_1} = 3\sqrt{3} \quad \left(\text{as } \frac{m_2}{m_1} = 3\right)$$ 1

$$\underline{\alpha = 79.11°}$$ 1

(ii) From (1) or (2),

$$T = \frac{m_1 g}{2\cos\alpha} \quad \text{or} \quad T = \frac{\sqrt{3}}{2}\,\frac{m_2 g}{\sin\alpha}$$ 1

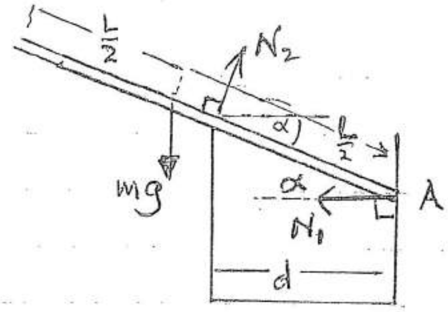
$$= \frac{(0.100)(9.81)}{2\cos(79.11°)} \qquad = \frac{\sqrt{3}}{2}\,\frac{(0.300)(9.81)}{\sin(79.11°)}$$

$$\underline{T = 2.60\text{ N}} \qquad \underline{T = 2.60\text{ N}}$$ 1

12

No matter which solution, 4 marks for the right diagram
2 marks for each right equation.

(b)



Correct forces 2

Introduce Normal reactions N_1 and N_2 as indicated in diagram and mass m of the straw

Resolving forces vertically

$$mg = N_2 \cos\alpha \qquad (1) \qquad 2$$

Taking moments about A

$$mg\left(\frac{L}{2}\right)\cos\alpha = N_2 \frac{d}{\cos\alpha} \qquad (2) \qquad 2$$

Dividing (1) by (2)

$$\frac{1}{\frac{L}{2}\cos\alpha} = \frac{\cos^2\alpha}{d} \qquad 1$$

$$\cos^3\alpha = \frac{2d}{L}$$

$$\cos\alpha = \sqrt[3]{\frac{2d}{L}} \qquad 1$$

8

Q5 17

(a) The path difference between rays reflected at the curved surface of the lens and the glass plate is $2t$. In addition there is a phase shift of π on reflection at the glass plate. 1

The optical path difference for constructive interference is thus

$$(n+\tfrac{1}{2})\lambda \qquad n=0,1,2,\ldots$$ 1

and for destructive interference

$$n\lambda \qquad n=0,1,2,\ldots$$ 1

Thus for constructive interference we require

$$\underline{2t = (n+\tfrac{1}{2})\lambda} \qquad n=0,1,2\ldots$$ 1

and for destructive interference

$$\underline{2t = n\lambda} \qquad n=0,1,2\ldots$$ 1

5

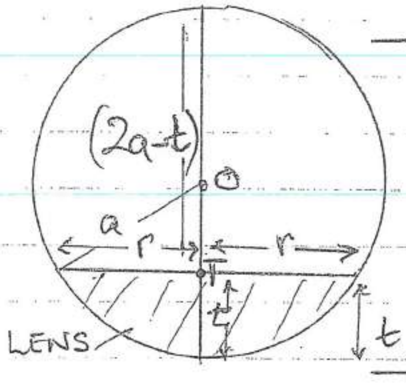
(b) In diagram

$$(OT)^2 = a^2 - r^2$$ 1

$$= (a-t)^2.$$ 1

$$\therefore\ a^2 - r^2 = a^2 - 2at + t^2$$

$$\underline{r^2 = 2at - t^2}$$ 1



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(c) Using approximation

$$r^2 = 2at \quad , \text{neglecting } t^2$$

the conditions for constructive and destructive interference, expressed in terms of r, are respectively

$$\frac{r^2}{a} = (n+\tfrac{1}{2})\lambda \qquad n=0,1,2,\ldots$$ 1

$$\frac{r^2}{a} = n\lambda \qquad n=0,1,2,\ldots$$ 1

2

Q 5

(d) Using diameter d, $\frac{d^2}{4a} = (n+\frac{1}{2})\lambda$ (A)

$$\frac{(0.582\times10^{-2})^2}{4a} = (n+\tfrac{1}{2})\lambda \qquad (1) \qquad 1$$

$$\frac{(1.36\times10^{-2})^2}{4a} = (n+20\tfrac{1}{2})\lambda \qquad (2) \qquad 1$$

Subtracting (1) from (2)

$$\frac{10^{-4}}{4a}\left(1.36^2 - 0.582^2\right) = 20\lambda = 20\times5.98\times10^{-7} \qquad 1$$

$$a = \frac{10^{-4}(1.942)(0.778)}{4\ (11.96\times10^{-6})}$$

$$\underline{a = 3.16\ \text{m}} \qquad (3) \qquad 1$$

4

Q5 (e) The initial radii are $r_1 = 0\cdot291$ cms and $r_2 = 0\cdot680$ cms
Introducing liquid of refractive index μ modifies the conditions for interference to

$$\mu\frac{r^2}{a} = (n+\tfrac{1}{2})\lambda \quad \text{and} \quad \frac{\mu r^2}{a} = (n+20\tfrac{1}{2})\lambda$$

So the new radii become r_1' and r_2' where

$$r_1'^2 = \frac{1}{\mu} r_1^2 \quad \text{and} \quad r_2'^2 = \frac{1}{\mu} r_2^2.$$

Substituting $\mu = 1\cdot33$ and $r_1 = 0\cdot291$ cms, $r_2 = 0\cdot680$ cms

$$r_1' = \frac{0\cdot291}{\sqrt{1\cdot33}} \text{ cms and } r_2' = \frac{0\cdot680}{\sqrt{1\cdot33}} \text{ cms}$$

$$r_1' = 0\cdot252 \text{ cms and } r_2' = 0\cdot590 \text{ cm}$$

So average speed of INCREASE in radii are V_1 and V_2 given by

$$V_1 = \frac{0\cdot252 - 0\cdot291}{5\cdot00} \text{ cms}^{-1} \ \& \ V_2 = \frac{0\cdot590 - 0\cdot680}{5\cdot00} \text{ cms}^{-1}$$

$$\underline{V_1 = -\ 0\cdot0078 \text{ cms}^{-1}} \ \& \ \underline{V_2 = -\ 0\cdot018 \text{ cms}^{-1}}$$

Interference rings decrease in radius.

6

MARK 20

Q6

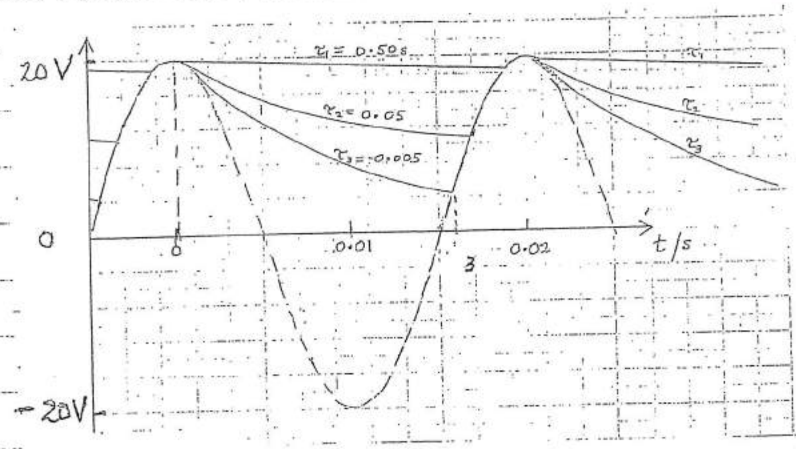
(a) Time constants given by $\tau = CR$ 1

$\tau_1 = (50\times10^{-6})10^{4}$
$= \underline{0.500\,s}$

$\tau_2 = (50\times10^{-6})10^{3}$
$= \underline{0.050\,s}$

$\tau_3 = (50\times10^{-6})10^{2}$
$=$ ~~0.00050s~~ 1
0.005 s 2

(b) GRAPH



Correct qualitative form of graphs for τ_1, τ_2 and τ_3 over one period; 2 marks for each curve } 6
1 mark for correct cosine curve 1
1 mark for correct labelling of axes 1
8

(c) T_D = ac period $- T_1$
$\underline{T_D = f^{-1} - T_1}$ (1)

The capacitor discharges until its voltage equals that of the ac source. For τ_3 this is is from $t=0$ to $t=$ 3 (see graph). At time T_1 the potential on capacitor $V_0 \exp(-T_1/RC)$ equals that due to ac source $V_0 \cos 2\pi(fT_1)$. Hence

$$V_0 \exp(-T_1/RC) = V_0 \cos 2\pi(fT_1)$$

or
$$\exp(-T_1/RC) = \cos 2\pi(fT_1) \quad (2)$$
} 3

Substituting $RC = 0.50\,s$ and $T_1 = 0.019125\,s$
LHS of (2) = 0.9625
RHS of (2) = 0.9625
Thus LHS=RHS, equation verified } 1

Q6

(c) % time diode conducting $= \frac{T_D}{f^{-1}}(100)$

$= \frac{f^{-1} - T_1}{f^{-1}}(100)$ from (1)

$= \left(1 - \frac{T_1}{f^{-1}}\right)100$

$= \left(1 - \frac{0.019125}{0.020000}\right)100$

$= 4.4\%$ 1

6

(d) At $T = 0.019125\,s$

$\frac{V}{V_0} = \exp(-0.019125/0.50) = 0.962.$ } 2

For $V_0 = 20\,V$ (peak to peak = 40 V)

$V = 19.24\,V$ 1

Thus peak to peak ripple voltage $= (20 - 19.24)\,V$

$= 0.76\,V$ 1

(Any result from 0.7 V to 0.8 V acceptable)

4

Q1

(a) RANDOM ERRORS

Not due to a systematic "mechanism" and with sufficient "readings" will average out to zero and give a correct measurement. The errors will have a gaussian distribution 1

EXAMPLES

A stop watch when stopped by hand, at a specific time, will introduce a random error due to one's reaction time.

An analogue ammeter reading may vary slightly due to the angle of observation. If a number of observations are made at varying angles, for a specific current, these errors will average out to zero. 1

SYSTEMATIC ERRORS

These are often due to a mechanism of which one is unaware. These errors do not average out to zero for repeated measurements but 'behave' in a systematic way 1

EXAMPLES

A stop watch that continually 'gains' time will give readings that are too high

An ammeter that has been incorrectly calibrated will consistently give incorrect readings that do not average out to the true value. 1

Other valid examples are acceptable.

Q7 (a) (i) $g = 9.8 \pm 1.0 \ ms^{-2}$ (Error relatively large compared with the value) 1

(ii)

Root mean square $l_{RMS} = \sqrt{\frac{1}{2}\left[(72.0)^2 + (65.0)^2\right]}$ m

$= \frac{1}{\sqrt{2}}(97.0)$ m

$l_{RMS} = 68.6$ m 1

6

(b) $T = 2\pi\sqrt{\frac{l}{g}}$ and $g = \frac{(2\pi)^2 l}{T^2}$

$\frac{\Delta g}{g} = 2\frac{\Delta T}{T} + \frac{\Delta l}{l}$

$= 2\frac{0.20}{100} + \frac{0.60}{100}$

$100\frac{\Delta g}{g} = 0.40 + 0.60$ 3

Max % error = 1.0 %

Alternative methods acceptable

Q7

(b) Alternatively one can evaluate max-error by substituting:

$$g = \frac{(2\pi)^2 l}{T^2} \quad (1)$$

Max error Δg:

$$g + \Delta g = \frac{(2\pi)^2 \, l(1+0.006)}{(T(1-0.002))^2}$$

Dividing by g from (1)

$$1 + \frac{\Delta g}{g} = \frac{1.006}{0.996}$$

Max. % error $100\frac{\Delta g}{g} = 1.0\%$

ALTERNATIVE

3

3

(c) Correctly drawn graph of $V = \log A + \alpha \log I$ from table of values 1
Axes labelled 1

~~Gradient within 2.25 ± 0.10 (One mark for the value and 2 marks for accuracy)~~ 3

ALTERNATIVELY
Gradient within 2.25 ± 0.25 but outside limits of 2.25 ± 0.10 (One mark for value, One mark for accuracy) 2

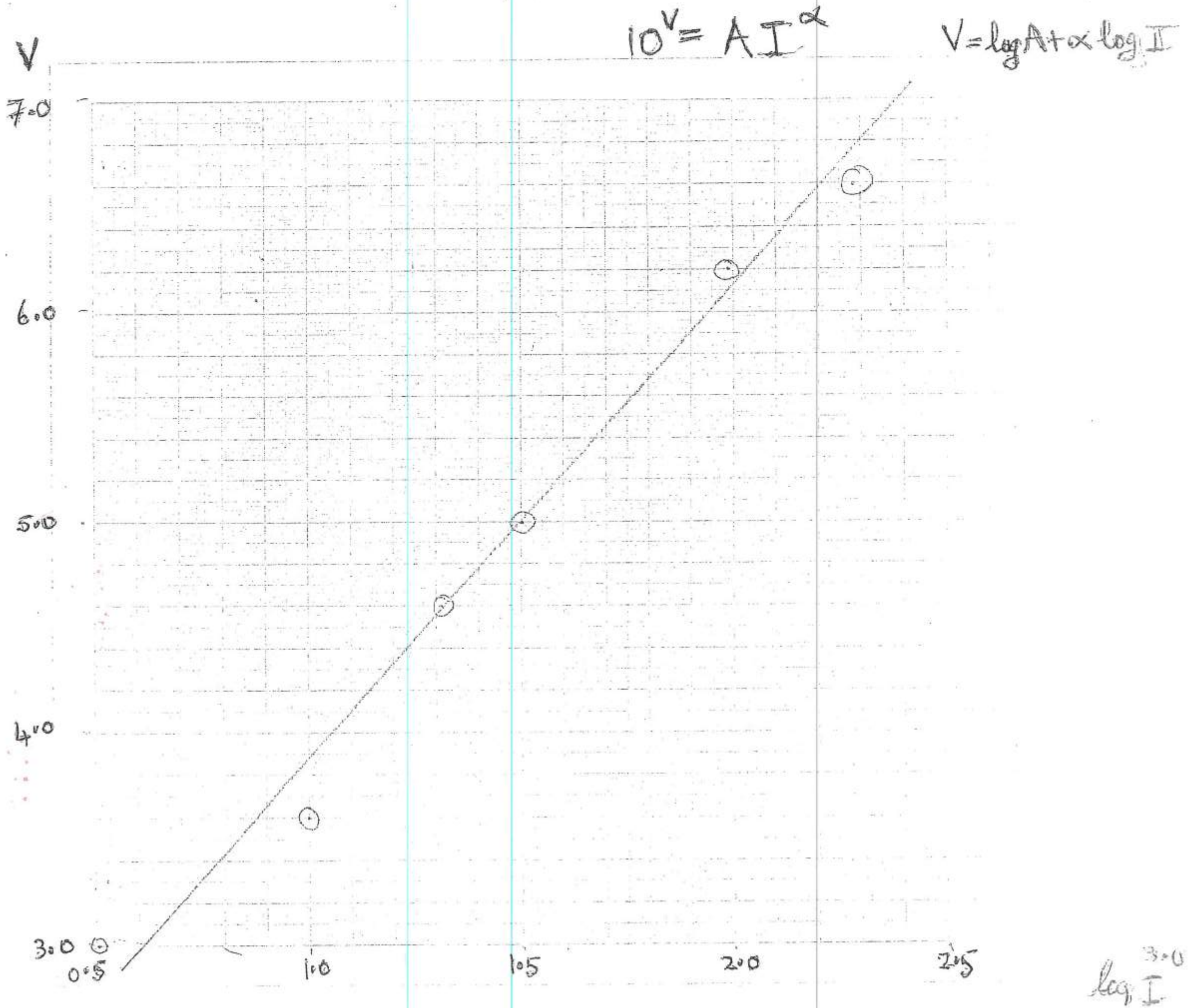
n	V	I	$\log_{10} I$
1	3.0	3.2	0.505
2	3.6	9.9	0.996
3	4.6	20.4	1.310
4	5.0	31.6	1.500
5	6.2	97.0	1.987
6	6.6	193	2.286

A obtained from graph or by substituting α info
$V = \log A + \alpha \log I$ for a particular (V, I)
Acceptable value $A = 42 \pm 4$ 2

7

* Mark breakdown: [2] for accurate graph + labels
[3] for gradient value within 2.25 ± 0.3
BUT minus [1] if students do NOT provide their own estimate of accuracy. [2] for value of intercept with estimate of accuracy.

/25



$10^V = A I^{\alpha}$
$V = \log A + \alpha \log I$
V
7.0
6.0
5.0
4.0
3.0
0.5
1.0
1.5
2.0
2.5
3.0
log I

Q7

(d)

	(1, 4)	(2, 5)	(3, 6)
α	$\frac{2.00}{0.995} = 2.01$	$\frac{2.60}{0.991} = 2.62$	$\frac{2.00}{\cancel{0.985}} = \cancel{2.03}$ 0.976 2.05
		$\alpha_M = \cancel{2.22}$ 2.23	
$\lvert\alpha - \alpha_M\rvert$	0.21	0.40	$\cancel{0.19}$ 0.178
$D = \text{Average}(\lvert\alpha - \alpha_M\rvert) = \cancel{0.27}$ 0.263			

1

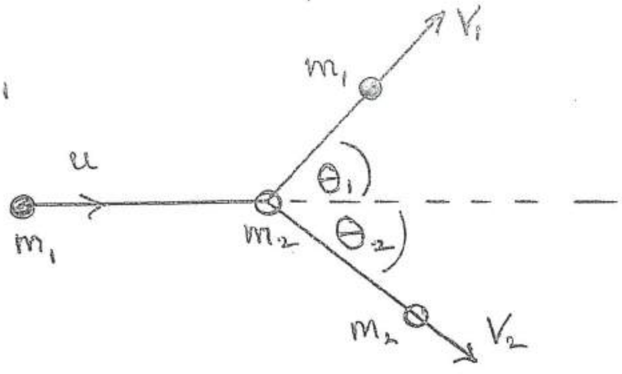
1

This is a quantitative method that does not depend on judging by eye the 'best' straight graph
(Any other sensible comment will gain the mark) 1

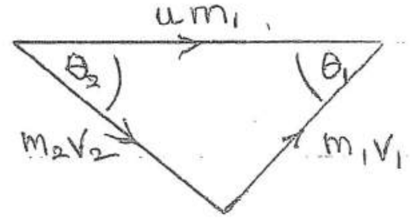
The average deviation, D, is greater than in (c) 1

4

Q8 (a) $m_2 = 4m_1$



TRIANGLE OF MOMENTA



Vector Diagram 1

Using sin rule,

$$\frac{m_1 V_1}{\sin\theta_2} = \frac{m_2 V_2}{\sin\theta_1}$$ 1

Thus

$$\frac{V_1}{V_2} = \frac{m_2 \sin\theta_2}{m_1 \sin\theta_1} = \frac{4\sin\theta_2}{\sin\theta_1} \quad (1)$$ 1

Also

$$\frac{m_1 V_1}{\sin\theta_2} = \frac{m_1 u}{\sin(180-\theta_1-\theta_2)}$$ 1

Thus

$$\frac{V_1}{u} = \frac{\sin\theta_2}{\sin(\theta_1+\theta_2)} \quad (2)$$ 1

5

(b) Final K.E., $T_F = \frac{1}{2}m_1 V_1^2 + \frac{1}{2}m_2 V_2^2 \quad (3)$ 1

Initial K.E. $T_I = \frac{1}{2}m_1 u^2$

From (3) & (1)

$$T_F = \frac{1}{2}m_1 V_1^2 + \frac{1}{2}m_2 V_1^2\left(\frac{m_1\sin\theta_1}{m_2\sin\theta_2}\right)^2$$

$$= \frac{1}{2}m_1 V_1^2\left[1 + \frac{m_1\sin^2\theta_1}{m_2\sin^2\theta_2}\right]$$

Q8 (b) Substituting from (2)

$$T_F = \tfrac{1}{2} m_1 u^2 \frac{\sin^2\theta_2}{\sin^2(\theta_1+\theta_2)} \left[1 + \frac{m_1 \sin^2\theta_1}{m_2 \sin^2\theta_2} \right]$$

$$= T_I \frac{\sin^2\theta_2}{\sin^2(\theta_1+\theta_2)} \left[1 + \frac{m_1 \sin^2\theta_1}{m_2 \sin^2\theta_2} \right] \quad 2$$

As $m_2 = 4m_1$

$$T_F = T_I \frac{\sin^2\theta_2}{\sin^2(\theta_1+\theta_2)} \left[1 + \frac{\sin^2\theta_1}{4\sin^2\theta_2} \right] \quad (4)$$

(i) Substituting $\theta_1 = \theta_2 = 60°$

$$T_F = T_I \frac{\sin^2 60°}{\sin^2(120°)} \left[1 + \frac{\sin^2 60°}{4\sin^2 60°} \right] \quad 1$$

As $\sin 60° = \sin 120°$ and $\sin 60 = \sqrt{3}/2$

$$T_F = T_I \left[1 + \frac{1}{4} \right] = \frac{5}{4} T_I \quad 1$$

Energy not conserved.

$$T_F = T_I \frac{\sin^2\theta_2}{\sin^2(\theta_1+\theta_2)} \left[1 + \frac{\sin^2\theta_1}{4\sin^2\theta_2} \right]$$

Q8 (b)
(ii) $\theta_1 = \theta_2 = 56°$

$$T_F = T_I \frac{\sin^2 56°}{\sin^2 112°} \left[1 + \frac{1}{4} \right] \quad 1$$

$$T_F = T_I \quad \text{to accuracy of } 2\times10^{-3} \quad 1$$

Energy conserved to within accuracy of 2×10^{-3}. 1

Q(8)(b)(iii) $\frac{v_1}{v_2} = 1 = \frac{4\sin\theta_2}{\sin 90^\circ}$

$\therefore \sin\theta_2 = \frac{1}{4}$ $(\sin 90^\circ = 1)$

From (4)

$$T_F = T_I \frac{\sin^2\theta_2}{\sin^2(90^\circ+\theta_2)}\left[1 + \frac{1}{4}\frac{1}{\sin^2\theta_2}\right]$$

Substituting,

$$T_F = T_I \frac{\left(\frac{1}{4}\right)^2}{\sin^2(90^\circ+\theta_2)}\left[1+\left(\frac{1}{4}\right)\frac{1}{(1/16)}\right]$$

$$= T_I \frac{5/16}{\sin^2(104 \cdot 48^\circ)}$$ 2

$$\underline{T_F = 0 \cdot 333\ T_I}$$ 1

Thus energy lost

12

(c)

$3\theta_1 = -8\theta_2$

Substituting into (4) with $\sin\theta_2 \sim \theta_2$, $\sin\theta_1 \sim \theta_1$

$$T_F = T_I \left(\frac{\theta_2}{\theta_1+\theta_2}\right)^2\left[1 + \frac{\theta_1^2}{4\theta_2^2}\right]$$ 1

$$= T_I \frac{(-3/8)^2}{[1+(-3/8)]^2}\left[1 + \frac{1}{4(-3/8)^2}\right]$$ 1

$$\underline{T_F = T_I}$$ Energy conserved 1

3

Q9

(a) $$E = -\frac{13.6}{n^2}\ \text{eV}$$

For transition from $n=3$ to $n=1$ $\quad \Delta E$ given by

$$\left.\begin{aligned}\Delta E &= -13.6\left(\frac{1}{3^2}-\frac{1}{1^2}\right)\\ &= \frac{8}{9}(13.6)\ \text{eV} = 12.1\ \text{eV}\end{aligned}\right\}\quad 1$$

Thus frequency ν given by

$$h\nu = 12.1\ (1.60\times10^{-19})\ \text{J} \qquad 1$$

$$\nu = \frac{12.1\ (1.60\times10^{-19})}{6.63\times10^{-34}}\ \text{Hz}$$

As $c=\lambda\nu$ $$\frac{1}{\lambda} = \frac{12.1\ (1.602\times10^{-19})}{6.63\times10^{-34}\ (3.00\times10^{8})}$$

$$\underline{\lambda = 1.03\times10^{-7}\ m} \qquad 1$$

$$\frac{}{3}$$

(b) Distance of Moon from Earth $= 3.84\times10^{8}$ m

Diameter of beam on Moon $= (3.84\times10^{8})(1.5\times10^{-3})$ m $\quad 1$

Area illuminated on Moon $= \pi\left(\frac{1}{4}\right)(3.84\times10^{8})^2(1.5\times10^{-3})^2\ m^2$

$= 2.61\times10^{11}\ m^2 \quad 1$

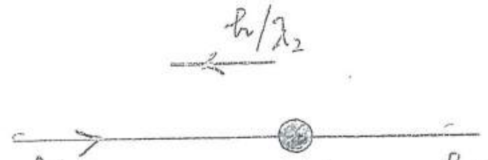
$$\left.\begin{aligned}&\text{Energy of a single photon} = h\nu\\ &\text{as } c=\lambda\nu \qquad = \frac{6.63\times10^{-34}\ (3.00\times10^{8})}{(590\times10^{-9})}\end{aligned}\right\}\quad 1$$

Number of photons, n, produced by laser per second given by

$$\left.\begin{aligned}n &= \frac{0.5\times10^{-3}\ (590\times10^{-9})}{(6.63\times10^{-34})(3.00\times10^{8})}\\ &= 1.48\times10^{15}\end{aligned}\right\}\quad 1$$

No. of photons arriving / unit area / sec. $= \underline{5.68\times10^{3}\ m^{-2}s^{-1}} \quad 1$

$$\frac{}{5}$$



Q9(c)

Momenta	h/λ_1	$m_e v$	h/λ_2
Energy	hf_1	$\frac{1}{2}m_e v^2$	hf_2

Conservation of momentum

$$\frac{h}{\lambda_1} = -\frac{h}{\lambda_2} + m_e v \quad (1)$$

Conservation of energy

$$hf_1 = hf_2 + \tfrac{1}{2} m_e v^2 \quad (2)$$

As "$f\lambda = c$"

$$\frac{h}{\lambda_1} = \frac{h}{\lambda_2} + \frac{1}{2c} m_e v^2 \quad (3)$$

Adding (1) and (3)

$$\frac{2hc}{\lambda_1} = \frac{1}{2} m_e v^2 + m_e v c \quad (4)$$

Now for 5.00 keV electrons,

$$\tfrac{1}{2} m_e v^2 = (5.00 \times 10^{3})(1.60 \times 10^{-19})$$

$$\tfrac{1}{2} m_e v^2 = 8.00 \times 10^{-16}\ \text{J} \quad (5)$$

Giving

$$v^2 = \frac{16.0 \times 10^{-16}}{9.11 \times 10^{-31}}$$

$$= 1.76 \times 10^{15}$$

$$v = 4.20 \times 10^{7}\ \text{ms}^{-1} \quad (6)$$

Substituting (5) and (6) into (4) with $c = 3 \times 10^{8}\ \text{ms}^{-1}$,

$$\frac{2hc}{\lambda_1} = 8.00 \times 10^{-16} + c(9.11 \times 10^{-31})(4.20 \times 10^{7})$$

$$= 8.00 \times 10^{-16} + 1.15 \times 10^{-14}$$

$$= 1.23 \times 10^{-14}$$

$$\lambda_1 = 3.23 \times 10^{-11}\ \text{m}$$

BPhO

British Physics Olympiad

BRITISH PHYSICS OLYMPIAD 2014-15

Round 1

Section 2

14th November 2014

Instructions

Questions: Only TWO of the eight questions in *Section 2* should be attempted.

Time: 1 hour 20 minutes on this section (approximately 40 minutes on each question).

Marks: The maximum mark for each of these questions is 20.

Answers
Answers and calculations can be written on loose paper or examination booklets. Graph paper and formula sheets are available.

Students should ensure their **name** and **school** is clearly written on all answer sheets.

Teachers' instructions
Section 1 and *Section 2* of *Paper 2* may be sat in one session of 2 hour 40 minutes.

Alternatively, the paper may be sat in two sessions on separate occasions; with 1 hour 20 minutes for *Section 1* and 1 hour 20 minutes for *Section 2*. If the paper is taken in two sessions on separate occasions, *Section 1* must be collected after the end of 1 hour 20 minutes and *Section 2* is to be handed out in the second session.

Q9(d) $eV_0 = h\nu - W$ where $V_0 = 1.32\,V$

$W = h\nu - eV_0$ $\lambda = 380\,nm$

W = Work function 2

$$= \frac{(6.63\times10^{-34})(3.00\times10^{8})}{(380\times10^{-9})} - (1.60\times10^{-19})(1.32)$$

$$= 5.23\times10^{-19} - 2.11\times10^{-19}\ J$$ 1

$$\underline{W = 3.12\times10^{-19}\ J}$$ 1

4