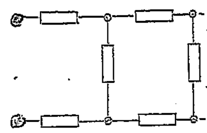
2014 BPhO PAPER 2 SOLUTIONS

Q1

(a) (i)

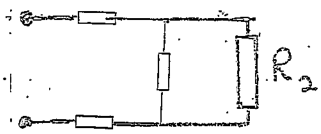


MARKS.

$$R_2 = 2R + \left(\frac{1}{R} + \frac{1}{3R}\right)^{-1}$$

$$= 2R + \frac{3}{4}R$$

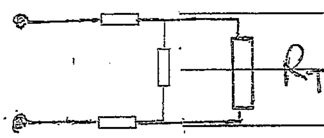
$$\underline{R_2 = 2\tfrac{3}{4}R}$$ 1



$$R_3 = 2R + \left(\frac{1}{R} + \frac{4}{11R}\right)^{-1}$$

$$\underline{R_3 = 2\tfrac{11}{15}R}$$ 1

(ii)



$$R_T = 2R + \left(\frac{1}{R} + \frac{1}{R_T}\right)^{-1}$$ 1

$$= 2R + \frac{RR_T}{(R + R_T)}$$

$$R_T^2 - 2RR_T - 2R^2 = 0$$ 1

Solving

$$R_T = \frac{2R \pm \sqrt{(2R)^2 + 4(2R^2)}}{2}$$

$$= R \pm \sqrt{3}\,R$$ 1

Only positive R_T acceptable, so

$$\underline{R_T = (\sqrt{3} + 1)R}$$ 1

[6]

Q1

(b) (i) For circular motion of satellite mass m, angular frequency ω,

$$\frac{GM_Em}{R^2} = mR\omega^2 \quad 1$$

$$= mR\left(\frac{2\pi}{T}\right)^2$$

$$G\left(\tfrac{4}{3}\pi R^3\rho\right)m = mR^3\left(\frac{2\pi}{T}\right)^2 \quad 1$$

$$\rho T^2 = \frac{3\pi}{G} \quad 1$$

This is independent of R.

(ii) Introduce spring constant k, then when in equilibrium

$$75g = k(0{\cdot}30)$$

$$k = \frac{75g}{0{\cdot}30} \quad 1$$

For SHM, $\omega^2 = \frac{k}{m}$ thus

$$\left(\frac{2\pi}{T}\right)^2 = \left(\frac{75g}{0{\cdot}30}\right)\frac{1}{75} \quad 1$$

Giving $$T^2 = \frac{(2\pi)^2(0{\cdot}30)}{g}$$

$$T = 2\pi\sqrt{\frac{0{\cdot}30}{g}}$$

$$T = 1{\cdot}10\,\text{s} \quad 1$$

[6]

3

Q1

(c)(i) If tube has length L and atmospheric pressure A, Boyle's law gives for 10 m depth,

$$AL = (A + 10\rho g)\tfrac{1}{2}L \qquad 1$$

$$\underline{A = 10\rho g}$$

If water fills 90% of tube at depth l, then Boyles law gives, for volume in $(\frac{1}{10})L$ of tube

$$AL = \left(\tfrac{1}{10}\right)L(A + l\rho g) \qquad 1$$

$$9A = l\rho g$$

$$l = \frac{9A}{\rho g} = \frac{90\rho g}{\rho g} \qquad \text{as } A = 10\rho g$$

$$\underline{l = 90\,m} \qquad 1$$

(ii) At 60° vertical height of mercury $\frac{1}{2}(950)$ mm. Applying Boyles law to volume above mercury in tube, with atmos. pressure A in mm of mercury

$$300(A - 700) = 50\left(A - \left(\tfrac{1}{2}\right)950\right) \qquad 2$$

$$6(A - 700) = A - 475$$

$$\underline{A = 745 \text{ mm}} \text{ of mercury} \qquad 1$$

[6]

Q1 (d)

$$V_{AV} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{\text{final dist.} - \text{initial dist.}}{\text{time taken}} \qquad \tfrac{1}{2}$$

$$= \frac{A\sin\left(5\left(\frac{\pi}{10}\right)\right) - 0}{\pi/10}$$

$$\underline{V_{AV} = \frac{10A}{\pi}} \qquad 1$$

$$f_{AV} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}} \qquad \tfrac{1}{2}$$

$$= \frac{5A\cos\left(5\left(\frac{\pi}{10}\right)\right) - 5A}{(\pi/10)} = \frac{10}{\pi}\,5A(\cos\pi/2 - 1)$$

$$\underline{f_{AV} = -\frac{50A}{\pi}} \qquad 1$$

[3]

Q1

(e) Total +ve and -ve charge for 1 gram is $\frac{1}{2}N_A e$ — 1

(molecular weight of hydrogen is 2·00)

Force between charges on Earth and Moon given by

$F = \frac{q_1 q_2}{4\pi\varepsilon_0 R^2}$ — each proton attracts each electron

$$F = \frac{\left(\frac{1}{2}N_A e\right)^2}{4\pi\varepsilon_0 R_{EM}^2} \qquad R_{EM} = \text{Earth-Moon dist.}$$ — 2

$$= \left[\frac{(6.02\times10^{23})(1.60\times10^{-19})}{2}\right]^2 \frac{1}{4\pi\varepsilon_0}\frac{1}{(3.84\times10^{8})^2}$$ — 1

$$\underline{F = 1.41\times10^{2}\,N}$$ — 1

[5]

(f) Radioactive equilibrium for uranium, U, and radium, R, requires

$$\lambda_R N_R = \lambda_U N_U$$ — 1

Now

$$\frac{N_R}{N_U} = \frac{3.23\times10^{-7}}{226} \Big/ \frac{1}{238} = \frac{3.23\times10^{-7}(238)}{226}$$ — 1

Now also

$$\lambda_R = \frac{\ln 2}{T_R} \quad \text{and} \quad \lambda_U = \frac{\ln 2}{T_U}$$

Thus

$$T_U = \frac{N_U T_R}{N_R}$$ — 1

$$= \frac{(226)(1600)}{3.23\times10^{-7}(238}$$ ~~years~~ — 1

Giving $\underline{T_U = 4.70\times10^{9}\text{ years}}$ — 1

5

Q1 For either beam, assuming particles have mass m

(g) Energy: $Ve = \frac{1}{2}mv^2$ (1) 1

Force $\frac{mv^2}{R} = Bev$ (2) 1

Giving $R = \frac{mv}{Be}$ (3) 1

Substituting for v from (1)

$$R = \frac{m}{Be}\sqrt{\frac{2Ve}{m}}$$ 1

Thus separation between the two beams ΔD given by

$$\Delta D = \frac{2}{B}\sqrt{\frac{2V}{e}}\left(\sqrt{m_D} - \sqrt{m_p}\right)$$ 1

m_D = deuteron mass
m_p = proton mass

Substituting gives

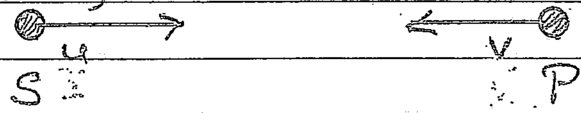
$$\Delta D = 2{\cdot}4 \text{ cm}$$ 1

[6]

Q1 Let $u = 15\,ms^{-1}\cos 75^\circ$ $v = 18\,ms^{-1}\cos 75^\circ$ $\nu = 200\,Hz$

(h) The source and observer are each moving towards each other at speeds of $15\cos 75^\circ$ and $18\cos 75^\circ$ ms^{-1}
ie u and v ms^{-1}
with numerical values of $3{\cdot}882\,ms^{-1}$ and $4{\cdot}659\,ms^{-1}$

Let us consider the Doppler effect in such a case where c is sound velocity



As source moving with velocity u, forward of source the distance between crests, the wavelength λ_s, appears to be given by

$$\lambda_s = \frac{c}{\nu} - \frac{u}{\nu} \qquad (1)$$

As observer moving towards source with speed v the velocity of waves relative to observer is $(c+v)$. This gives rise to frequency

$$\nu_p = \frac{(c+v)}{\lambda_s}$$

Therefore from (1)

$$\nu_p = \frac{(c+v)}{(c-u)}\,\nu$$

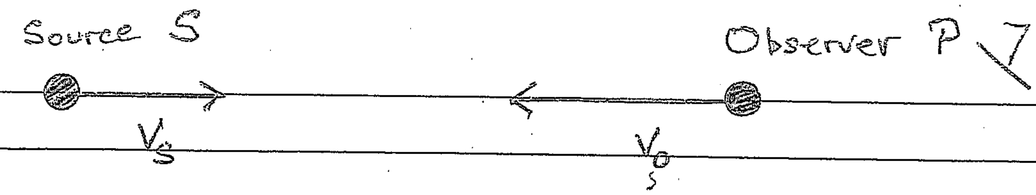
This is the frequency heard by the observer P.

Substituting for c, v and u

$$\nu_p = \left(\frac{331 + 4{\cdot}659}{331 - 3{\cdot}882}\right) 200 \quad Hz$$

$$\underline{\nu_p = 205\ Hz}$$

Q1
(h)



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As source moving with velocity V_s in same direction as sound, distance between crests of sound waves, wavelength λ_s, is

$$\lambda_s = \frac{c}{f} - \frac{V_s}{f} \qquad (1) \qquad 1$$

An observer moving towards source with velocity V_o, the velocity of sound waves relative to observer is $(c+V_o)$. This produces frequency

$$f_o = \frac{c+V_o}{\lambda_s} \qquad 1$$

Therefore from (1)

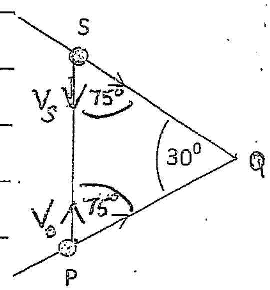
$$f_o = \frac{(c+V_o)}{(c-V_s)}\, f \qquad 1$$

Now $V_s = 15\cos 75^\circ$

and $= 3{\cdot}88\ \text{ms}^{-1}$ 1

$V_o = 18\cos 75\ \text{ms}^{-1}$

$= 4{\cdot}66\ \text{ms}^{-1}$ 1



So

$$f_o = \left(\frac{331 + 4{\cdot}66}{331 - 3{\cdot}88}\right) 200 \qquad 1$$

$$f_o = 205\ \text{Hz} \qquad 1$$

[8]

Q1 (c)

Volume of copper $= \dfrac{250 \times 10^{-3}}{8.93 \times 10^{3}}\ m^3$

upthrust on copper $= \left(\dfrac{\cancel{250 \times 10^{-6}}}{8.93}\right) 10^{3} g\ N$ (density water $10^3 kg/m^3$) 1

Thus Downthrust on water $= \dfrac{250 \times 10^{-3}}{8.93}\ g\ N$

$= \dfrac{250 \times 9.81 \times 10^{-3}}{8.93}\ N$

$= 0.275\ N$

Weight of 0.300 kg $= 0.300\ g\ N$

$= 2.94\ N$ 1

Reading on scales $= 2.94 + 0.275\ N$

$= \underline{3.21\ N}$ 1

[3]

8

Q1

(j) Let Q_1 and Q_2 be charges on C_1 and C_2, respectively, after switch closed.

Conservation of charge requires $\quad Q_1 + Q_2 = Q_0 \quad$ (1) $\quad$ 1

Potentials, V_1 and V_2, on capacitors are given by

$$V_1 = \frac{Q_1}{C_1} \quad \text{and} \quad V_2 = \frac{Q_2}{C_2} \qquad 1$$

As $V_1 = V_2$

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_0 - Q_1}{C_2} \quad \text{from (1)}$$

$$Q_1 = \frac{C_1}{C_1 + C_2} Q_0 \qquad 1$$

and

$$Q_2 = \frac{C_2}{C_1 + C_2} Q_0 \qquad 1$$

Initial energy $\quad U_0 = \frac{1}{2} \frac{Q_0^2}{C_1} \quad$ 1

Final energy $\quad U_f = \frac{1}{2} \frac{Q_1^2}{C_1} + \frac{1}{2} \frac{Q_2^2}{C_2} \quad$ 1

$$U_f = \frac{1}{2} \frac{Q_0^2}{(C_1 + C_2)} \qquad 1$$

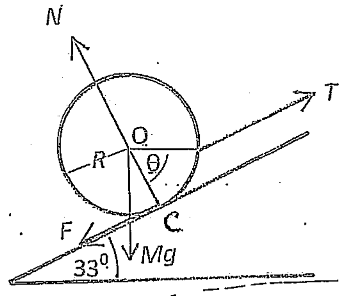
Thus $U_f < U_0$

Energy lost by heating $\quad$ 1

[8]

10

Q1
(k)



(i)

Resolving along slope $\quad T - F - Mg\sin 33^\circ = 0 \quad$ (1) $\quad$ 1

Resolving ⊥ʳ slope $\quad N - Mg\cos 33^\circ = 0 \quad$ (2) $\quad$ 1

As $F = \mu N$ in (1) and substituting for N from (2), gives $\quad$ 1

$$T = Mg(\sin 33^\circ + 0.420\cos 33^\circ) \quad 1$$

Substituting for Mg,

$$\underline{T = 44.0 \text{ N}} \quad 1$$

(ii) Taking moments about O,

$$R\,T\cos\theta = FR$$

as $N = Mg\cos 33^\circ$ $\quad$ 1

Sub^{n} for F $\quad \cos\theta = \frac{\mu Mg\cos 33^\circ}{T} \quad$ 1

$$= \frac{(0.420)(5.00)(9.81)\cos 33^\circ}{44.0} \quad 1$$

$$= 0.393 \quad 1$$

Giving

$$\underline{\theta = 66.9^\circ} \quad 1$$

[10]

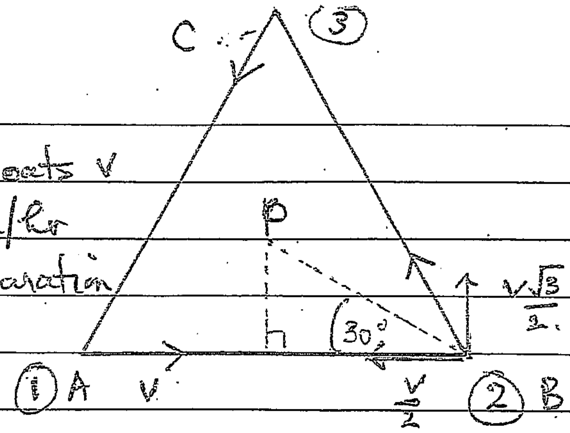
Q1

(ℓ) Speed of boats v

$v = 30$ km/hr

Initial separation

50 km.



METHOD 2

By symmetry they will meet at the centre of the triangle P. Resolving velocity towards P and perpendicular to C with velocity $v\frac{\sqrt{3}}{2}$ towards P and have to travel a distance in this direction, BC, of d given by (see diagram) 3

$$d = \frac{50}{2}\left(\frac{2}{\sqrt{3}}\right) = 50\left(\frac{\sqrt{3}}{3}\right) = 28.9 \text{ km}$$ 2

So time taken

$$T = 50\frac{\sqrt{3}}{3} \Big/ 30\left(\frac{\sqrt{3}}{2}\right) = \frac{10}{9} = 1\tfrac{1}{9} \text{ hrs}$$ 3

Consequently $D = \frac{300}{9} = 33.3$ km (as in previous result)

MARKS

Q1 (m) For air in barrel at atmospheric pressure, P_a, which holds k mols of air in volume V_b.

$$P_a V_b = kRT$$

Sub^g values: $(1.0\times10^5)(9.0\times10^{-5}) = kRT$ ① 1

After n strokes of the pump the tyre holds (nk) moles in volume V_T (T = tyre) at pressure P_T. Thus

$$P_T V_T = nkRT$$ 1

Sub^g values

$$(3.0\times10^5)(1.2\times10^{-3}) = nkRT \quad ②$$ 1

Substituting for kRT from ① into ②

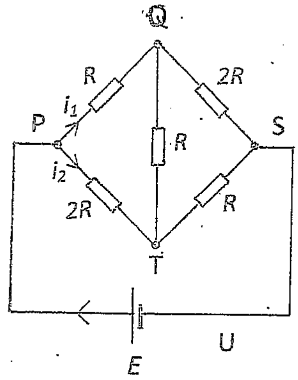
$$n = \frac{(3.0\times10^5)(1.2\times10^{-3})}{(1.0\times10^5)(9.0\times10^{-5})}$$

$$= 40$$ 1

4

Q2

(a)



Reversing the polarity produces an identical circuit to the former one. So current in $SQ = i_2$ and that in ST is i_1 as the reversal only reverses the directions of the currents in the arms

Thus $\quad i_{QS} = i_2$ 1

and $\quad i_{TS} = i_1 \quad$ in original circuit 1

(b) As sum of current entering any junction is equal to the sum leaving, at Q we have

$$i_{QT} = i_1 - i_2$$ 1

and at S

$$i_{SUP} = i_2 + i_1$$ 1

(c) For closed path PQTP,

$$i_1 R + (i_1 - i_2)R - 2i_2 R = 0$$ 1

or $\quad 2i_1 - 3i_2 = 0 \quad$ or $2i_1 = 3i_2$ 1 (1)

For closed path PTSUP, from (1),

$$E = 2Ri_2 + i_1 R \quad \text{i.e. } E = \frac{7}{2} i_2 R = \frac{7}{3} i_1 R \quad (2)$$

(d) $$R_{PS} = \frac{2Ri_2 + Ri_1}{(i_1 + i_2)}$$ i.e. $\frac{\text{voltage across PTS}}{\text{current entering at P}}$ 1

$$R_{PS} = \frac{7}{5} R$$ from (1) 1

From (1) and (2),

$$i_1 = \frac{3}{7}\frac{E}{R} \quad \text{and} \quad i_2 = \frac{2}{7}\frac{E}{R}$$ 2

Q2 (d) The resistance across PS is independent of any resistance in arm SVP. So if the internal resistance of E is 3R the resistance across PS remains $\frac{7}{5}R$. | 1

(e) The equation around closed path PQTP becomes

$$i_1 R + (i_1 - i_2) X - 2 i_2 R = 0 \qquad 1$$

$$i_1 (R+X) = i_2 (2R+X)$$

$$i_2 = \left(\frac{R+X}{2R+X}\right) i_1 \qquad (3) \qquad 1$$

Then $$R_T = \frac{2R i_2 + R i_1}{i_1 + i_2} \qquad 1$$

Substituting from (3) $$R_T = \frac{2R\left(\frac{R+X}{2R+X}\right) + R}{1 + \left(\frac{R+X}{2R+X}\right)} \qquad 1$$

$$= R\,\frac{4R+3X}{3R+2X}$$

$$= R\,\frac{4 + 3\left(\frac{X}{R}\right)}{3 + 2\left(\frac{X}{R}\right)} \qquad 1$$

When $\left(\frac{X}{R}\right) = 0$, $R_T = \frac{4}{3}R$ ½

$\frac{X}{R} \to \infty$, $R_T \to \frac{3}{2}R$ ½

Thus R_T ranges from $1.33\ldots R$ to $1.50R$. 1

[20]

Q3

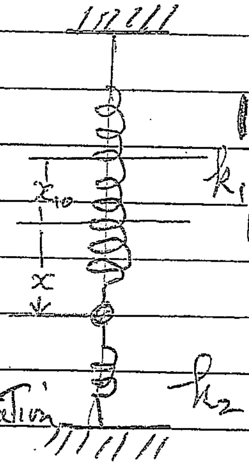
(a) If the spring with spring constant k_1 is stretched by an amount x, then $(\frac{1}{n})$th of the spring is stretched by $(\frac{x}{n})$. As the forces in the whole spring and $(\frac{1}{n})$th of the spring are the same we require 2

$$k_1 x = k_2 \left(\frac{x}{n}\right) \qquad 1$$

$$\text{or} \quad k_2 = n k_1 \qquad 1$$

(b) At equilibrium

$$k_1 x_{10} = mg$$
$$x_{10} = \frac{mg}{k_1} \qquad (1) \qquad 1$$



(c) If mass displaced by x from its equilibrium position, the equation of motion is, if a is the acceleration,

$$ma = -k_1(x+x_{10}) - k_2 x + mg \qquad 3$$
$$= -(k_1+k_2)x. \quad \text{from (1)} \qquad 1$$

If $k_2 = nk_1$,
$$= -(n+1)k_1 x$$

This is SHM with angular frequency

$$\omega = \sqrt{\frac{(n+1)k_1}{m}} \qquad 1$$

(d)
$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}k_1(x+x_{10})^2 + \tfrac{1}{2}k_2x^2 - mgx \qquad 4$$

$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}x^2(k_1+k_2) + k_1 x x_{10} - mgx + \tfrac{1}{2}k_1 x_{10}^2 \qquad 1$$

From (1),

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}(k_1+k_2)x^2 + \tfrac{1}{2}k_1 x_{10}^2 \qquad 1$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}k_1(n+1)x^2 + \tfrac{1}{2}k_1 x_{10}^2$$

where $k_2 = nk_1$. This depends only on the variable x^2 and v^2.

(e) For the horizontal system $x_{1,0} = 0$ and the equation of motion is unchanged. So it performs SHM with angular frequency given by

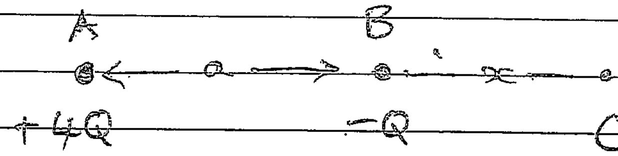
$$\omega^2 = \frac{(n+1)k_1}{m}$$

Q4

(a) (i) A line in the field which at every point is tangential to the electric field vector 1

(ii) A surface which has constant electric potential at all points on the surface in the electric field 1

(b)



The neutral point C is a point where the electric field is zero.

The condition for the magnitudes of the fields from A and B to be equal is

$$\frac{4Q}{4\pi\varepsilon_0(a+x)^2} = \frac{Q}{4\pi\varepsilon_0 x^2} \qquad (1) \qquad 1$$

By symmetry C will lie along the line through AB. 1

From (1) $\quad 4x^2 = (a+x)^2$

$$3x^2 - 2ax - a^2 = 0$$

Giving $\quad x = \dfrac{2a \pm \sqrt{4a^2+4\cdot 3a}}{6}$

$$= a\left(\tfrac{1}{3}\right)(1 \pm 2) \qquad 1$$

Thus there are two roots to the quadratic equation a and $-\frac{1}{3}a$.

$(-\frac{1}{3}a)$ corresponds to a non-zero electric field where the electric field vectors from A and B add

The other solution $x = a$, corresponds to a single point C where there is zero electric field 1

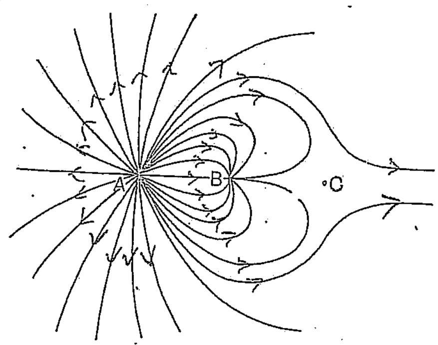
(c) $\dfrac{4Q}{4\pi\varepsilon_0 r_A} - \dfrac{Q}{4\pi\varepsilon_0 r_B} = \text{constant} = c \text{ say}$

OR $\quad \dfrac{4}{r_A} - \dfrac{1}{r_B} = c'$ where c' a constant

Difference values of the constant produce different surfaces. 2

Q4

(d) (i) Field Lines



Award marks for:

Symmetry $\frac{1}{2}$

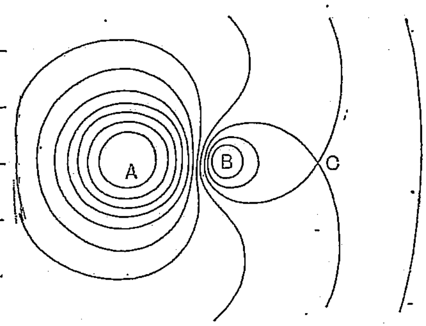
Neutral point C $\frac{1}{2}$

Arrows $\frac{1}{2}$

General shape of field lines $\frac{3}{2}$

} 3

(ii) Equipotential Surfaces



Award Marks for:

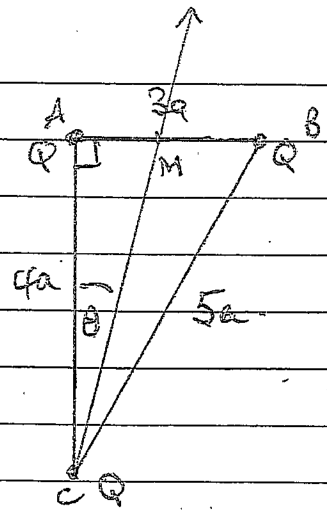
Symmetry $\frac{1}{2}$

Neutral point C $\frac{1}{2}$

General Form of shape of equipotentials 2

} 3

Q4
(e) (ii)



Potential at M, V_M, is given by

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{\frac{3}{2}a} + \frac{Q}{\frac{3}{2}a} + \frac{Q}{\sqrt{(4a)^2 + (\frac{3}{2}a)^2}}\right]$$

$$= \frac{Q}{4\pi\varepsilon_0 a}\left[\frac{4}{3} + \frac{2}{\sqrt{73}}\right]$$

$$= \frac{Q}{4\pi\varepsilon_0 a}\left[\frac{4}{3} + \frac{2\sqrt{73}}{73}\right]$$

$$V_M = \frac{1.57\,Q}{4\pi\varepsilon_0 a}$$

Electric Field Vector $\underline{E}$

Fields from charges at A and B cancel
Only contribution arises from C

$$E = \frac{Q}{4\pi\varepsilon_0\left[(4a)^2 + (\frac{3}{2}a)^2\right]} \quad \text{along CM}$$

$$= \frac{Q}{4\pi\varepsilon_0 a^2\left(16 + \frac{9}{4}\right)}$$

$$= \frac{Q}{4\pi\varepsilon_0 a^2}\left(\frac{4}{73}\right)$$

$$\underline{E} = \frac{Q\,(0.0548)}{4\pi\varepsilon_0 a^2} \quad \text{at angle } \theta \text{ to CA}$$

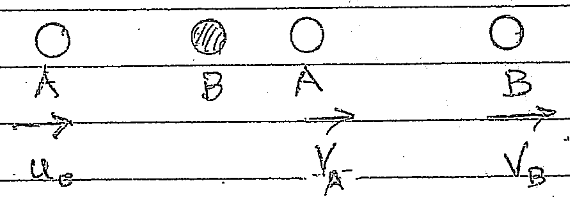
where

$$\tan\theta = \frac{3a}{2}\Big/4a = \frac{3}{8} \qquad \theta = A\hat{C}M$$

$$\theta = 20.6^\circ$$

Q 5

(a)



Momentum Conservation

$$m u_0 = m V_A + m V_B$$

i.e. $$u_0 = V_A + V_B \qquad (1)$$ 1

Energy Conservation

$$\tfrac{1}{2} m u_0^2 = \tfrac{1}{2} m V_A^2 + \tfrac{1}{2} m V_B^2$$

i.e. $$u_0^2 = V_A^2 + V_B^2 \qquad (2)$$ 1

Substituting for V_B from (1) into (2)

$$u_0^2 = V_A^2 + (u_0 - V_A)^2$$
$$= u_0^2 - 2u_0 V_A + 2V_A^2$$
$$V_A(V_A - u_0) = 0$$

So

either $V_A = 0$ or $V_A = u_0$

As $V_A = u_0$ corresponds to no collision, we require 1

$$V_A = 0$$

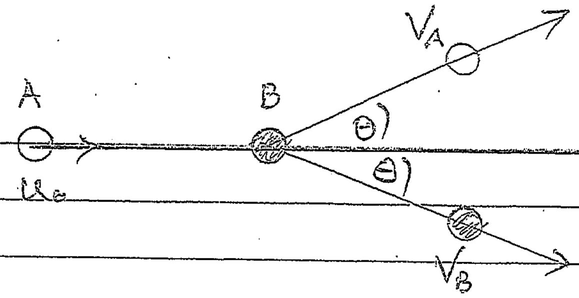
This gives from (1),

$$V_B = u_0$$ 1

Thus after impact $V_A = 0$ (A at rest) and $V_B = u_0$.

Q: 5

(b)



(i) Energy Conservation

$$\tfrac{1}{2}mu_0^2 = 8.00 \qquad \text{ie } u_0 = \frac{4}{\sqrt{m}} \qquad (1) \quad 1$$

$$\tfrac{1}{2}mu_0^2 = \tfrac{1}{2}mV_A^2 + \tfrac{1}{2}mV_B^2 + 2.00 \qquad 1$$

So
$$8.00 = \tfrac{1}{2}m(V_A^2+V_B^2) + 2.00$$
$$6.00 = \tfrac{1}{2}m(V_A^2+V_B^2)$$
$$12.00 = m(V_A^2+V_B^2) \qquad (2) \quad 1$$

(ii) Momentum Conservation horizontally

$$mu_0 = mV_A\cos\theta + mV_B\cos\theta = m\cos\theta(V_A+V_B) \qquad (3) \quad 1$$
$$u_0 = \cos\theta\,(V_A+V_B) \qquad 1$$

Momentum Conservation vertically

$$0 = mV_A\sin\theta - mV_B\sin\theta \qquad 1$$
$$\text{ie} \quad V_A = V_B \qquad (4) \quad 1$$

(iii) From (3) and (4)

$$\cos\theta = \frac{u_0}{2V_A} \qquad 1 \quad (5)$$

From (2) and (4) $\quad V_A = \sqrt{\frac{6}{m}} \quad \& \quad V_B = \sqrt{\frac{6}{m}} \qquad 2 \quad (6)$

Thus, from (1) and (6), (5) becomes

$$\cos\theta = \frac{2}{\sqrt{6}} = \sqrt{\frac{2}{3}} \qquad 1$$

$$\underline{\theta = 35.3^\circ} \qquad 1$$

(iv) Change in vector momentum of A $= m\underline{V}_A - m\underline{u}_0$

Conservation of vector momentum $\; m\underline{u}_0 = m\underline{V}_A + m\underline{V}_B$

Thus $\quad m\underline{V}_A - m\underline{u}_0 = -m\underline{V}_B$

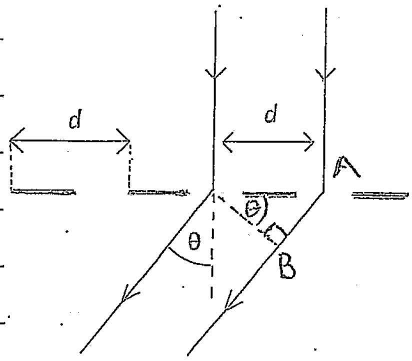
From (6) $\quad = -m\sqrt{\frac{6}{m}}\,\hat{V}_B \quad$ where $\hat{V}_B$ is unit vector along $\underline{V}_B$

$$= -\sqrt{6m}\,\hat{V}_B$$

Thus momentum change has magnitude $\underline{\sqrt{6m}}$ and direction is opposite to $\underline{V}_B$.

4

Q.6
(a)



Path difference AB = $d\sin\theta$ 1

(i) Condition for constructive interference for wavelength λ

$$d\sin\theta = n\lambda \quad (1) \quad n \text{ integer}$$ 1

(ii)

$$d\sin\theta = (n+\tfrac{1}{2})\lambda \quad (2)$$ 1

3

(b) Red light has larger wavelength than violet light so the values of θ, with same n, that produce constructive interference, from (1), are larger for red light than violet light. Consequently it is possible for the n th constructive interference fringe for violet light to coincide with the (n−1) th fringe for red light. This has occurred here. ← ($1\frac{1}{2}$) Explanation

The n=3 red fringe has coincided with the n=2 violet fringe. The order of the fringes are:

1st	violet	$n_V = 1$	($\frac{1}{2}$)
2nd	red	$n_R = 1$	($\frac{1}{2}$)
3rd	violet	$n_V = 2$	($\frac{1}{2}$)
4th	red + violet	$n_R = 2$ & $n_V = 3$	($\frac{1}{2}$)

So

5th	violet	$n_V = 4$	($\frac{1}{2}$) (3)

4

Q6

(c) For the composite line

$$\sin\theta = \frac{n_R \lambda_R}{d} = \frac{n_V \lambda_V}{d}$$ 2

Thus

$$= \frac{2\lambda_R}{d} = \frac{3\lambda_V}{d}$$ 1

Now

$$d = \frac{10^{-3}}{500}\ \text{m} \quad \text{and} \quad \theta = 43{\cdot}6^\circ.$$ 1

So substituting $\lambda_V = \frac{d\sin\theta}{3} = \underline{460\ \text{nm}}$ 1

$$\lambda_R = \frac{d\sin\theta}{2} = \underline{690\ \text{nm}}$$ 1

6

(d) The 5th line from (3) corresponds to $n_V = 4$ 9

So the corresponding θ is given by

$$\sin\theta = \frac{4\,(460)10^{-9}}{2\times10^{-6}}$$ 1

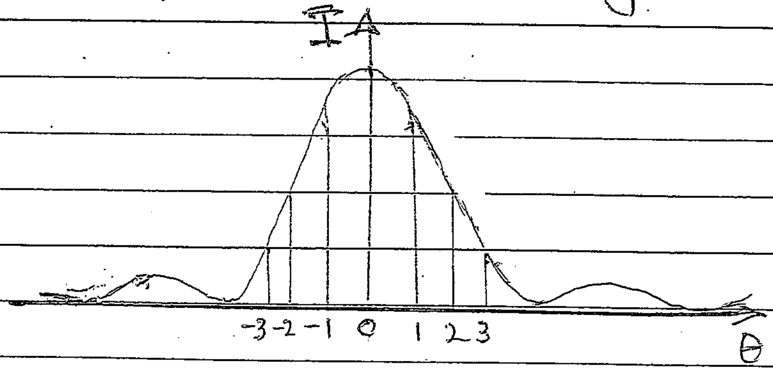
$$= 0{\cdot}920$$

$$\underline{\theta = 66{\cdot}9^\circ}$$ (n_V, λ_V) 1

4

(e) Correct envelope of intensity, I 2

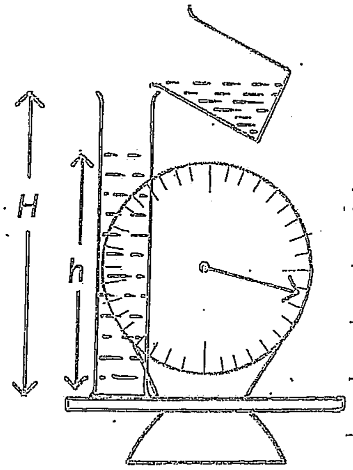
Indication of 3 diffraction lines of orders 1, 2 and 3 1



3

Q7

(a)



Force due to water impacting on water in measuring cylinder is equal to the rate of change of momentum on falling through a height $(H-h)$ gives a force $\rho v\sqrt{2g(H-h)}$ and thus 1

$$w = W_{MC} + \rho v\sqrt{2g(H-h)} + hA\rho g \qquad 2 \quad (1)$$

where $\rho A\rho g$ is the weight of liquid in measuring cylinder

(b)

$$w = W_{MC} + \rho v\sqrt{2g}\sqrt{H-h} - (H-h)A\rho g + HA\rho g$$

$$= (W_{MC}+HA\rho g) - A\rho g\left[(\sqrt{H-h})^2 - \frac{v\sqrt{2g}}{gA}\sqrt{H-h}\right]$$

Substituting $y^2 = (H-h)$ 3

$$w = (W_{MC}+HA\rho g) - A\rho g\left[y^2 - by\right] \qquad (2)$$

where $2b = \frac{v}{A}\sqrt{\frac{2}{g}}$ (3)

$$w = (W_{MC}+HA\rho g) - A\rho g\left[y^2 - 2by + b^2\right] + A\rho g b^2$$

$$= (W_{MC}+HA\rho g + A\rho g b^2) - A\rho g\left[y-b\right]^2 \qquad (4) \quad 1$$

w is a maximum when $y = b = \frac{V}{A}\sqrt{\frac{1}{2g}}$ and w has the value (5)

$$W_{max} = (W_{MC} + HA\rho g + A\rho g b^2) \qquad 1 \quad (6)$$

Q7

(b) $$\underline{w_{max} = W_{MC} + HA\rho g + \frac{v^2\rho}{2A}}$$ 1 (7)

(c) At w_{max}, $y_{max} = \sqrt{H - h_{max}} = b = \frac{v}{A}\sqrt{\frac{1}{2g}}$

Squaring

$$H - h_{max} = \frac{v^2}{A^2}\frac{1}{2g}$$

Weight of liquid in cylinder at w_{max}, $= \rho g A h_{max}$ 1

$$= \rho g A\left(H - \frac{v^2}{A^2 2g}\right)$$

Difference Δ between (7) and $W_{MC} + \rho g A\left(H - \frac{v^2}{2A^2 g}\right)$

$$\Delta = +\rho g A\left(\frac{v^2}{2A^2 g}\right) + \frac{v^2\rho}{2A}$$

$$\underline{\Delta = \frac{\rho v^2}{A}}$$ 1

(d) Time t_1 for liquid to initially fall height H before impacting is given by

$$H = \tfrac{1}{2}g t_1^2 \quad \text{ie} \quad t_1 = \sqrt{2H/g} \quad \& \; w = W_{MC} \quad (8)$$

From $t = 0$ to $t = t_1$ $w = W_{MC}$ $\frac{1}{2}$

As subsequently $h = \frac{v}{A}t$ $t_1 \leq t \leq t_E$ (8a)

we can express (4) in terms of time t

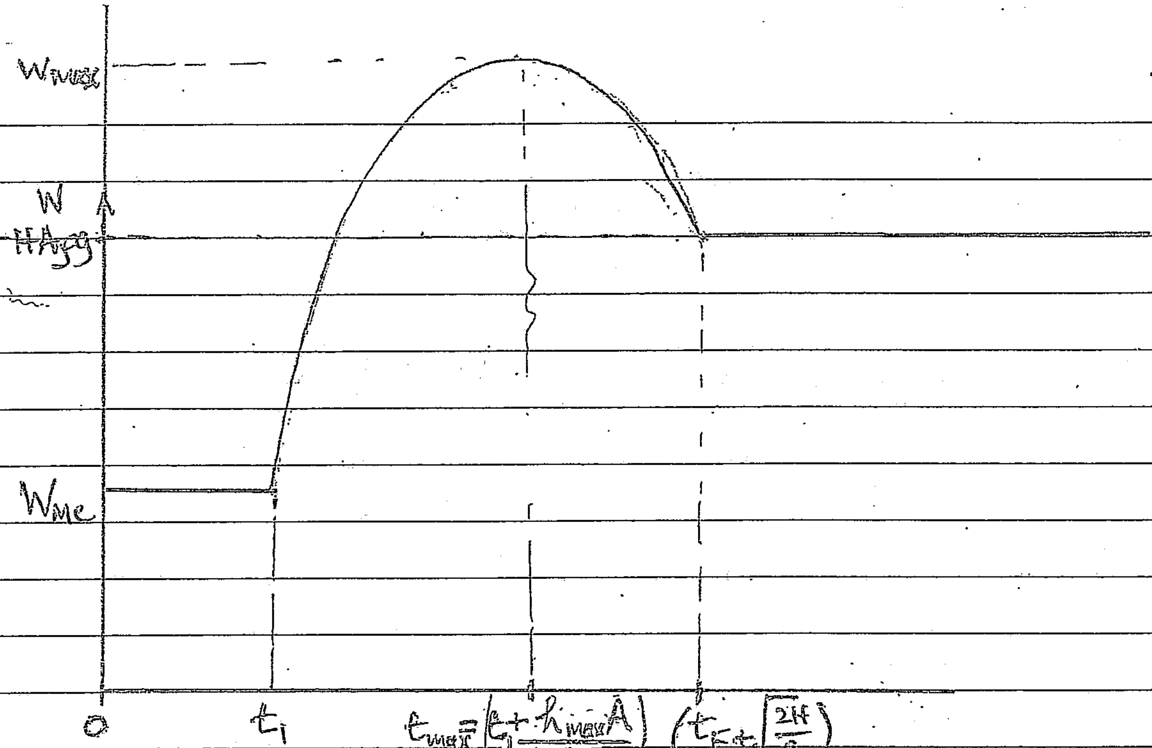
$$w = (W_{MC} + HA\rho g + A\rho g b^2) - A\rho g\left[\sqrt{H - \tfrac{v}{A}(t - t_1)} - b\right]^2$$

$$t_1 \leq t \leq t_E + \sqrt{\frac{2H}{g}}$$ $2\frac{1}{2}$ (9)

$$w = W_{MC} + HA\rho g \qquad t \geq t_E + \sqrt{\frac{2H}{g}}$$ 1 (10)

Q7

(e)



From (8a),

W_{max} occurs at time $t_{max}=\left(t_1+\frac{Ah_{max}}{V}\right)$.

MARKING GRAPH.

Correct initial linear portion	$\frac{1}{2}$
Final linear portion	$\frac{1}{2}$
Curve with maximum	1
(W_{MC}, t_1) point on graph	$\frac{1}{2}$
$(W_{MC}+HA\rho g)$, $\sqrt{\frac{2H}{g}}$) point on graph	1
Maximum point (W_{max}, t_{max})	$\frac{1}{2}$

4