8/10/09

# 2012 BPhO Paper 2 Solutions

Q1

(a)(i) $R_{AC} = \left(\frac{1}{2R} + \frac{1}{R} + \frac{1}{2R}\right)^{-1}$ 1

$R_{AC} = \frac{1}{2}R$ 1

(ii) $R_{AB}$ = R in parallel with $\{R_{BC}$ + $R_{AC}$ in parallel with $R_{AD} + R_{DC}\}$

= R in parallel with $\left\{R_{BC} + \left(\frac{1}{R} + \frac{1}{2R}\right)^{-1}\right\}$

= R in parallel with $\left\{R + \frac{1}{3}2R\right\}$ 1

= R in parallel with $\frac{5}{3}R$

$= \left(\frac{1}{R} + \frac{3}{5R}\right)^{-1}$

$R_{AB} = \frac{5}{8}R$ 1

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(b)(i) Ionization energy $= -E_1$

$= 2{\cdot}16 \times 10^{-18}$ J 1

(ii) Frequency of $H_\alpha$ line, $\nu$, given by

$h\nu = 2{\cdot}16\times10^{-18}\left(\frac{1}{4} - \frac{1}{9}\right) = 2{\cdot}16\times10^{-18}\left(\frac{5}{36}\right)$ 1

Wavelength $\therefore \lambda = \frac{c}{\nu} = \frac{3{\cdot}00\times10^{8}\,(6{\cdot}63\times10^{-34})}{2{\cdot}16\times10^{-18}\,(0{\cdot}13889)}$ 1

$\lambda = 6{\cdot}63\times10^{-7}$ m 1

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(c) Top block will collapse relative to lower block if its moment about edge with lower block is greater than zero.

i.e. if $-mg.(6\times10^{-2}) > 0$ 2

Consequently it is stable relative to lower block.

The two blocks will tilt about edge of table if their moment about

Q1 (c) this edge is greater than zero. This moment is

$$(x-12)mg + (x-6)mg \qquad 1$$
$$= (2x-18)mg$$

Consequently collapse occurs if

$$(2x-18)mg > 0 \qquad 1$$

$$\text{i.e.} \quad x > 9 \text{ cm} \qquad 1$$

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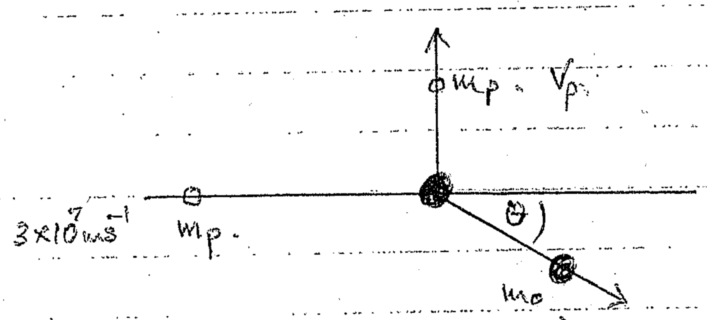
(d)




Let mass of oxygen nucleus be $m_o$

Let $u_p = 3\cdot00\times10^7 ms^{-1}$, $V_{oh}$ and $V_{ov}$ be the horizontal and vertical components of the oxygen nucleus's velocity components and $V_p$, be the ~~vertical component of the~~ velocity of proton after collision.

Conservation of horizontal momentum

$$u_p m_p = V_{oh} m_o \quad \text{i.e.} \quad V_{oh} = u_p \frac{m_p}{m_o} \qquad (1)$$

Conservation of vertical momentum

$$0 = V_p m_p - V_{ov} m_o \quad \text{i.e.} \quad V_{ov} = V_p \frac{m_p}{m_o} \qquad (2)$$

Energy conservation

$$\frac{1}{2} m_p u_p^2 = \frac{1}{2} m_p V_p^2 + \frac{1}{2} m_o \left( V_{oh}^2 + V_{ov}^2 \right)$$

Sub. from (2),

$$= \frac{1}{2} m_p V_p^2 + \frac{1}{2} m_o \left( V_{oh}^2 + V_p^2 \frac{m_p^2}{m_o^2} \right)$$

$$u_p^2 = V_{pv}^2 + \frac{m_o}{m_p} \left( V_{oh}^2 + V_{pv}^2 \frac{m_p^2}{m_o^2} \right)$$

$$V_{pv}^2 \left[ 1 + \frac{m_p}{m_o} \right] = u_p^2 - \frac{m_o}{m_p} V_{oh}^2$$

From (1)

$$= u_p^2 - u_p^2 \cdot \frac{m_p}{m_o}$$

Q1 (d) $V_{pv} = u_p \cdot \sqrt{\dfrac{1-\frac{m_p}{m_o}}{\left(1+\frac{m_p}{m_o}\right)}}$ 3

Now $\dfrac{m_p}{m_o} = 0.06523$ so

$$V_{pv} = 3.00 \times 10^7 \sqrt{\frac{0.93476}{1.06523}}$$

$\underline{V_{pv} = 2.81 \times 10^7 \ ms^{-1}}$ 1

Direction of oxygen nucleus to horizontal, $\theta$.

$V_{oh} = u_p \left(\dfrac{m_p}{m_o}\right) = 3.00 \times 10^7 (0.06523) = 1.96 \times 10^6 \ ms^{-1}$

$V_{ov} = V_{pv}\left(\dfrac{m_p}{m_o}\right) = 2.81 \times 10^7 (0.06523) = 1.83 \times 10^6 \ ms^{-1}$ 2

$\therefore \tan\theta = \dfrac{1.83}{1.96} = 0.937$

$\underline{\theta = 43.1^\circ}$ 1

$\underline{V_o^2 = V_{oh}^2 + V_{ov}^2}$

$V_o^2 = \left[(1.96)^2 + (1.83)^2\right] 10^{12}$ 1

$\underline{V_o = 2.68 \times 10^6 \ ms^{-1}}$ 1

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Q1 (e) Volume of wreck $= \frac{10^4}{8\times10^3}$ m$^3$. 1

Mass of water displaced by wreck $= \frac{10^4}{8\times10^3}\times10^3$ kg

Up thrust on submerged wreck $= \frac{10^4}{8\times10^3}\times10^3\,(9.81)$ N 1

If $\Delta l$ is extension of cable when wreck is lifted el, then Young's modulus gives

$$5\times10^{10} = \frac{\frac{10^4}{8\times10^3}\times10^3\,(9.81)}{5\times10^{-4}} \Big/ \left(\frac{\Delta l}{10}\right)$$ 2

$$\Delta l = \frac{(10)\,\frac{10^4}{8\times10^3}\times10^3\,(9.81)}{(5\times10^{10})(5\times10^{-4})}$$

$$= \frac{9.81}{2\times10^3}$$

$$= 4.9 \text{ mm}$$ 1

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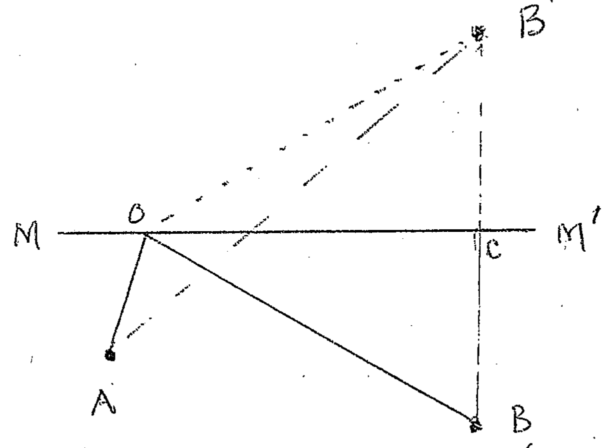
Q1 (f)




(i) As B' is reflection of B, OB' = OB (right angled $\Delta$s BOC & B'OC congruent) 1

Thus path AOB = AOB' 1

(ii) AOB': has minimum length when O along AB' (straight line joining AB') 2

Thus AOB " " " " " " " 1

When this occurs $\hat{AOM} = \hat{BOM'}$, so angle of incidence = angle of reflection i.e. for reflected ray AOB. 1

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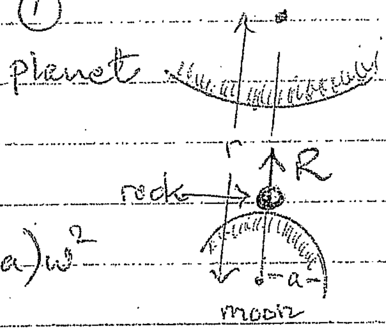
Q1 (g) Circular Orbit:

(i) $$\frac{GMm}{r^2} = mr\omega^2$$ 1

$$GM = r^3\omega^2 \quad (1)$$




(ii) Equilibrium of rock on moon requires

$$\frac{GM\mu}{(r-a)^2} - \frac{Gm\mu}{a^2} + R = \mu(r-a)\omega^2$$ 2

$\mu$ is the mass of the rock.

If rock to be lifted off moon by gravitational pull of planet 1

$R<0$ ie

$$\frac{GM\mu}{(r-a)^2} - \frac{Gm\mu}{a^2} - \mu(r-a)\omega^2 > 0$$ 1

From (1)

$$G\frac{M}{m}\left[\frac{m}{(r-a)^2}\right] - \frac{Gm}{a^2} - \frac{(r-a)GM}{r^3} > 0$$

$$\frac{M}{m}\left[\frac{1}{(r-a)^2} - \frac{(r-a)}{r^3}\right] > \frac{1}{a^2}.$$

$$\frac{M}{m} > \frac{1}{a^2}\,\frac{(r-a)^2 r^3}{r^3-(r-a)^3}.$$ 3

$$\frac{M}{m} > \frac{r^3}{a^2}\,\frac{(r-a)^2}{3r^2a-3ra^2+a^3}.$$

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Q1 (h)

$$(4.80)\,400 = (35-15)\,40\,(1.6\times10^3)\times10^{-6}\times 860 + H \quad (1)$$ 1

$$(4.90)\,400 = (35-15)\,20\,(1.6\times10^3)\times10^{-6}\times 860 + (35-15)\,20\,(2\times10^3)\,10^{-6}s + H \quad (2)$$ 1

From (1)

$$1920 = 1.100\times10^3 + H \quad (3)$$ 1

From (2)

$$1960 = 0.5504\times10^3 + 0.800s + H \quad (4)$$ 1

Subtracting (3) from (4)

$$40 = 0.800s - 0.5504\times10^3$$

(i) $$s = 738\ \mathrm{J\,kg^{-1}\,K^{-1}}$$ 1

From (3) $$H = 1920 - 1100 \approx 820\ \mathrm{J}$$ 1

6

Q1 (i) If $V_{xd}$ is the $x$ component of the velocity of electron at $x=d$, then at time $t$,

$$V_{xd} = V_x + ft = V_x + \frac{Ee}{m_e}t \quad (A) \qquad 1$$

As $V_x << \frac{Eet}{m_e}$,

$$V_{xd} \cong \frac{Eet}{m_e} \quad (B) \qquad 1$$

If $\theta$ angle final trajectory makes with $x$-axis and $V_y$ y-component of vel.

$$\tan\theta = \frac{V_y}{V_{xd}} \quad (C) \qquad 1$$

If the electrons appear to come from a point of horizontal distance $X$ from the point where they emerge from the field E

$$\tan\theta = \frac{V_y t}{X} \qquad 1$$

$$X = V_y t / \tan\theta$$

$$= V_{xd}t \quad \text{from (C)} \qquad 1$$

$$X = \left(\frac{Ee}{m_e}\right)t^2 \quad \text{from (B)} \qquad 1$$

As "$s = ut + \frac{1}{2}ft^2$"

$$d = \frac{1}{2}\left(\frac{Ee}{m_e}\right)t^2 \qquad 1$$

Thus

$$X = 2d$$

and electrons appear to originate from $x = -d$ 1

8

Q1 (j) Binding energy per nucleon $\mathcal{E}$ given by

$[2(1.0080 + 1.0087) = 4.0334]$

$$\mathcal{E} = (4.0334 - 4.0026)u/4$$ 1

$$= 0.0308u/4$$

$$= 0.0077(930)\ \text{MeV}$$ } 1

$$= 7.16\ \text{MeV}$$ 1

3

(k) (i) $c = \lambda f = 2 \times 8.0 \times 10^{-2} \times 2 \times 10^{3} = 320\ \text{ms}^{-1}$ 1

(ii) $320 = 1600(\lambda)$

$\lambda = 20\ \text{cm}$

Distance between adjacent nodes $= \frac{1}{2}\lambda = 10\ \text{cm}$ 1

(iii) In (i) length of tube L given by

$L = n\,8$ (1)

In (ii)

$L = (n-1)10$ (2)

Solving (1) and (2) for $n$,

$n = 5$ } 1

Giving $L = 40\ \text{cm}$ 1

(iv) The next frequency occurs when $n = 3$ giving

$L = 3\left(\frac{\lambda}{2}\right)$ 1

Sub$^{g}$ for L, $40 = \frac{3}{2}\lambda$

$\lambda = \frac{80}{3}\ \text{cm}$

This corresponds to $f = 320/(80/3)$

ie $f = 1200\ \text{Hz}$ 1

6

Q1

(l) Weight balances rate of change of momentum: $mg = \frac{d}{dt}(mv)$ 1

$$(810)\,9.81 = v^2(\pi 4^2)(1.20)$$ 1

$$\underline{V = 11.5\ \text{ms}^{-1}}$$ 1

Power $P = \frac{1}{2}(\pi 4^2)v^3(1.2)$ 1

$$= 9.6\pi v^3 = 9.6\pi(11.5)^3$$

$$\underline{P = 4.59 \times 10^4\ \text{W}}$$ 1

5

(m)(i) Probability of not decaying is $\frac{N(T)}{N_0} = \frac{1}{2}$ $\frac{1}{2}$

$\therefore$ Probability of decaying $= 1 - \frac{1}{2} = \frac{1}{2}$ $\frac{1}{2}$ } 1

(ii) Probability of not decaying $P_N = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

OR $P_N = \frac{N(3T)}{N_0} = e^{-3T\lambda}$ 1

$$= e^{-3\ln 2} \quad \text{as } \lambda = \frac{\ln 2}{T}$$

$$= \frac{1}{8}$$ 1

Thus probability of decaying $= 1 - \frac{1}{8}$

$$\underline{= \frac{7}{8}}$$ 1

4

(n) EITHER

$$\frac{\delta\rho}{\rho} = \frac{\delta m}{m} + \frac{\delta l_1}{l_1} + \frac{\delta l_2}{l_2} + \frac{\delta l_3}{l_3}$$

$$= \frac{0.03}{1.5} + \frac{1}{80} + \frac{1}{50} + \frac{1}{30}$$

$$= 0.020 + 0.0125 + 0.020 + 0.0333$$

$$= 0.0858$$

$$\underline{\frac{\delta\rho}{\rho} \simeq 0.09}$$

OR extreme and mean values

$$\frac{\delta\rho}{\rho} = \frac{1.53}{(79)(49)(29)} - \frac{1.50}{(80)(50)(30)} \Big/ \frac{1.50}{(80)(50)(30)}$$

$$\underline{\frac{\delta\rho}{\rho} = 0.09}$$

1

2

Q2 (a) pd around the circuit gives

(i) $(24.0 - 12.6) = 11.4 = 5(R + 1.10)$ 1

$R = \frac{11.4}{5} - 1.10$

$R = 1.18\,\Omega$ 1

(ii) Current $I_1$ $(24.0 - 12.6) = (0.9 + 1.0 + 0.1)I$ 1

$I = \frac{11.4}{2} = 5.70\,A$

$V_1 = (24.0 - 5.7) = 18.3\,V$ 1

$V_2 = 12.6 + (5.7)(0.10)$

$V_2 = +13.2\,V$ 1

5

(b) current $i$ through $E_1$

$i(R_1 + 10) = 2.0$ (1) 1

Also as no current thro' $E_2$, $iR_1 = 1.5$ ie $i = \frac{1.5}{R_1}$ 1

Substituting into (1)

$1.5 + 10i = 2.0$

$i = 0.05\,A$

$\therefore R_1 = \frac{1.5}{0.05} = 30\,\Omega$ 1

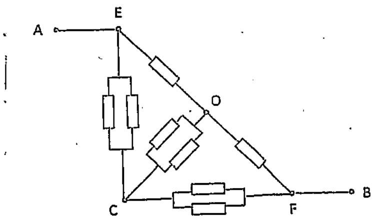
3

Q2 (c)(i) Circuit symmetrical about AB so potentials $V_C = V_D$ 1
(Alternative answers acceptable)

(ii) No change in currents in the arms or potentials as C and O at ~~the same~~ potential 1

(iii) Simplification by joining C to D
Two resistances in parallel give resultant resistance

$$\left(\frac{1}{R}+\frac{1}{R}\right)^{-1} = \frac{R}{2}$$ 1



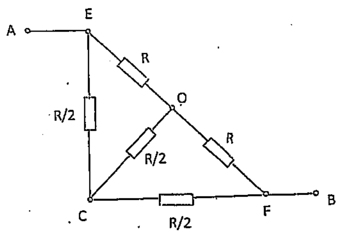




Diagram:
with correct resistors 1

(iv) pd across CO is zero as circuit forms a 'balanced' Wheatstone Bridge OR potentials at O and C equal as reversing pd across AB give same circuit, by symmetry, as original situation with pd across AB not reversed and current in CO cannot be reversed, so it must be zero 1
(Alternative solutions acceptable)

(v) $R_{AB}$ due to $\left(\frac{R}{2}+\frac{R}{2}\right)$ in parallel with $(R+R)$ 1

ie $R_{AB} = \left(\frac{1}{R}+\frac{1}{2R}\right)^{-1}$

$\underline{R_{AB} = \frac{2}{3}R}$ 1

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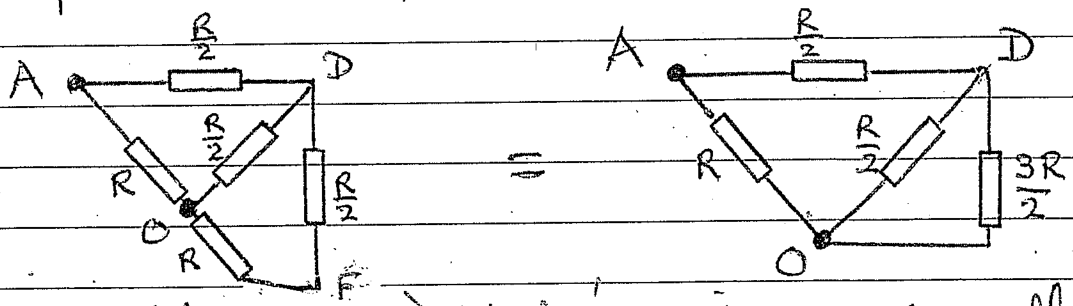
Q2 (d)

By symmetry C and D at same potential 1

Joining C and D and 'adding' resistance in parallel the circuit reduces to that below.



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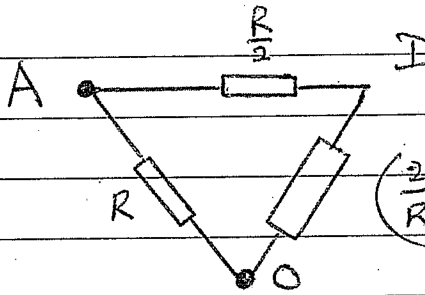
Simplifying by 'adding' resistances in parallel the circuit becomes



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$$\left(\frac{2}{R} + \frac{2}{3R}\right)^{-1} = \frac{3R}{8}$$

Finally 'adding' the resistances in parallel

$$\frac{1}{R_{AO}} = \left(\frac{R}{2} + \frac{3R}{8}\right)^{-1} + \frac{1}{R}$$

$$= \frac{8}{7R} + \frac{1}{R} = \frac{15}{7R}$$

$\therefore\ R_{AO} = \dfrac{7R}{15}$ 1

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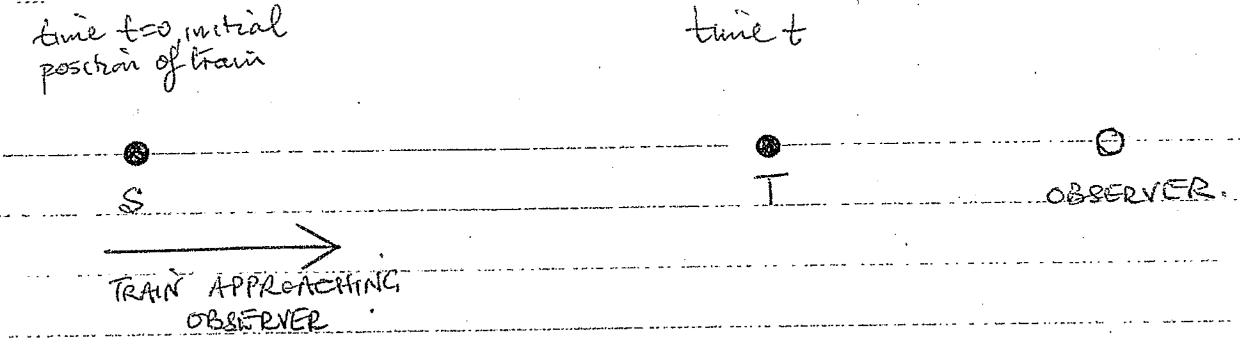
Q3 (a)




If train initially at S and reaches T in time t, the separation of crests of sound waves seen by observer viewing approaching train

$$\left(\frac{V_s}{f_0} - \frac{u}{f_0}\right) \quad \text{(wave appear 'compressed')}$$

$$= \lambda_a \quad \text{where } \lambda_a \text{ wavelength seen by observer}$$

$$\therefore \lambda_a = \frac{V_s - u}{f_0}$$

4 FOR CORRECT EXPLANATION

For observer viewing departing train, wavelength $\lambda_d$, $(u \to -u)$

$$\lambda_d = \frac{V_s + u}{f_0}$$

Thus for approaching train, frequency $f_a$ given by

$$f_a = \frac{V_s}{\lambda_a} = \frac{V_s}{(V_s - u)} f_0 \qquad 1$$

For departing train frequency $f_d$ given by

$$f_d = \frac{V_s}{\lambda_d} = \frac{V_s}{(V_s + u)} f_0 \qquad 1$$

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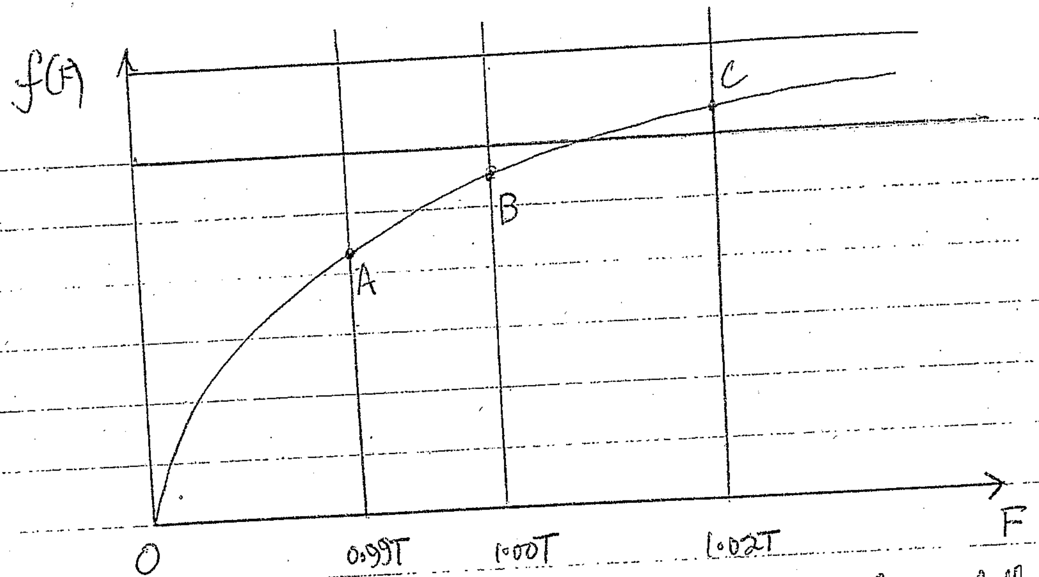
Q3 (b)
(i)




Data shows decreasing values of b; b being the modulus of the difference [of] two frequencies.

$f_0$ cannot below frequency at A as b would increase with increasing [F]
$f_0$ cannot be between frequencies at A and B as $b_3$ would have to be greater than $b_2$
It could however lie between the frequencies at B and C or above C

4

(ii) From the graph,

$$b_1 = f_0 - f(0.99T) = 3.30$$
$$b_2 = f_0 - f(T) = 2.00$$

Thus

$$f(0.99T) = f_0 - 3.30 \quad (1)$$
$$f(T) = f_0 - 2.00 \quad (2)$$

1
1

(iii) (1) and (2) can be written as

$$\frac{1}{2\ell}\sqrt{\frac{0.99T}{m}} = f_0 - 3.30$$

$$\frac{1}{2\ell}\sqrt{\frac{T}{m}} = f_0 - 2.00 \quad (A)$$

So

$$(f_0 - 2.00)\sqrt{0.99} = f_0 - 3.30$$

$$f_0(1-\sqrt{0.99}) = 2.00\sqrt{0.99} - 3.30$$

$$f_0 = 261\ \text{Hz}$$

Q3(b)(iv) We need to determine if $f_0 - f(1.02T)$ is +ve or −ve.

Now $\quad f_0 - f(T) = f_0 - (f_0 - 2) \quad$ from (A) or (2)

Q3(b) (iv) We need to determine if $f_0 - f(1.02T)$ is +ve or −ve.

Now
$$f_0 - f(1.02T) = f_0 - \sqrt{1.02}\,(f_0 - 2)$$

$$\left(\text{as } f(1.02T) = \sqrt{1.02}\, f(T) = \sqrt{1.02}\,\frac{1}{2\ell}\sqrt{\frac{T}{m}} \quad \text{from A} \right.$$
$$\left. = \sqrt{1.02}\,(f_0 - 2) \right)$$
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Thus
$$f_0 - f(1.02T) = f_0\left(1 - \sqrt{1.02}\right) + 2\sqrt{1.02}$$

Substituting for $f_0$,
$$= -261(0.00995) + 2.0199$$
$$= -0.58$$

ie $f_0$ lies between B and C on graph

$$f(1.02T) = f_0 + 0.58$$

(v)
$$f(F) = \frac{1}{2\ell}\sqrt{\frac{F}{m}}$$
$$\frac{\Delta f(F)}{f(F)} = \frac{\Delta \ell}{\ell} = \frac{2}{261} = 0.77\,\%$$
2

OR
$$\frac{\Delta f}{f(F)} = \frac{\dfrac{-1}{2(\ell+\Delta\ell)}\sqrt{\dfrac{T}{m}} + \dfrac{1}{2\ell}\sqrt{\dfrac{T}{m}}}{\dfrac{1}{2\ell}\sqrt{\dfrac{T}{m}}}$$
$$= \frac{\dfrac{1}{\ell} - \dfrac{1}{\ell+\Delta\ell}}{\dfrac{1}{\ell}} = 1 - \frac{\ell}{\ell+\Delta\ell}$$
$$\approx \frac{\Delta\ell}{\ell} = 0.77\,\%$$
{2}

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Q4 (a) If temperature changes from 15C to 20C, $l$ will change from $l_{15}$ to $l_{20}$ where

$$l_{20} = l_{15}(1+5\alpha)$$

As $$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\delta T = 2\pi\sqrt{\frac{l_{15}(1+5\alpha)}{g}} - 2\pi\sqrt{\frac{l_{15}}{g}}$$
$$= 2\pi\sqrt{\frac{l_{15}}{g}}\left[\sqrt{1+5\alpha} - 1\right]$$
$$= 2\pi\sqrt{\frac{l_{15}}{g}}\left[\tfrac{1}{2}(5\alpha)\right]$$

As $T = 2\pi\sqrt{\frac{l_{15}}{g}} = 1$, $$= \tfrac{1}{2}5\alpha$$

$$\underline{\delta T = 4.75\times10^{-5}\,s}$$

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ALTERNATIVELY $$\frac{\delta T}{T} = \frac{1}{2}\frac{\delta l}{l_{15}}$$

As $T=1$ $$\delta T = \tfrac{1}{2}5\alpha$$
$$= 4.75\times10^{-5}\,s.$$

4

Now $1\text{ week} = 7\times24\times60\times60\,s = 6.048\times10^{5}\,s$

Thus it will lose $(4.75\times10^{-5})(6.048\times10^{5})\,s$
$$\underline{= 29\,s}$$

1

5.

(b) $$g = \frac{GM}{R^2}$$
$$\frac{\delta g}{g} = -2\frac{\delta R}{R}$$

As $\delta R = 20m$, $$= -\frac{2(20)}{R} \quad (1)$$

Now $$T = 2\pi\sqrt{\frac{l}{g}}$$
$$\frac{\delta T}{T} = -\frac{1}{2}\frac{\delta g}{g}$$

From (1) $$= \frac{20}{R}$$

As $T=1$ $$\delta T = \frac{20}{6.38\times10^{6}}$$
$$= 3.135\times10^{-6}\,s$$

OR $$\frac{\delta g}{g} = \frac{\frac{GM}{(R+20)^2} - \frac{GM}{R^2}}{\frac{GM}{R^2}}$$
$$= -\left[1 - \frac{R^2}{(R+20)^2}\right]$$
$$\approx -\frac{2(20)}{R}$$

$$\frac{\delta T}{T} = \frac{2\pi\sqrt{\frac{l}{g+\delta g}} - 2\pi\sqrt{\frac{l}{g}}}{2\pi\sqrt{\frac{l}{g}}}$$
$$= \left(\frac{1}{\sqrt{g+\delta g}} - \frac{1}{\sqrt{g}}\right)\Big/\frac{1}{\sqrt{g}}$$
$$= \left(\sqrt{\frac{g}{g+\delta g}} - 1\right) = \left(1+\frac{\delta g}{g}\right)^{-1/2} - 1$$
$$= -\frac{1}{2}\frac{\delta g}{g} + \dots \text{ Binomial Theorem}$$
$$= \frac{20}{R} \text{ etc.}$$

3

(Alternatively evaluate T and T+$\delta$T to obtain $\delta$T)

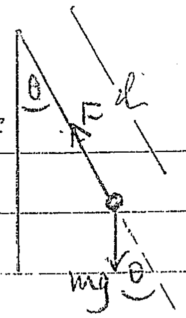
Thus it will lose $(3.135\times10^{-6})(6.048\times10^{5})\,s$
$$\underline{\simeq 1.9\,s}$$

1

4

24

(c) Resolving along direction F
$\dot{\theta}$ is the angular velocity,




$F - mg\cos\theta = ml\dot{\theta}^2$ (note ang. vel. $\omega = \dot{\theta}$)

$F = mg\cos\theta + ml\dot{\theta}^2$ 1

As $\theta$ and $\dot{\theta}$ vary, F will vary. 1

(i) At maximum amplitude $\theta_M$, $\dot{\theta} = 0$. 1

So $F_M = mg\cos\theta_M < mg$ 1

(ii) At $\theta = 0$ $F_0 = mg + ml\dot{\theta}^2 > mg$ 1

5

(d) For SHM with angular frequency $\omega$ and amplitude A, maximum retardation, a, occurs when

$a = \omega^2 A$ 1

The sand will leave the membrane when $a = g$ ie

$g = \omega^2 A$ 1

or $g = (2\pi f)^2 A$ where f is the frequency 1

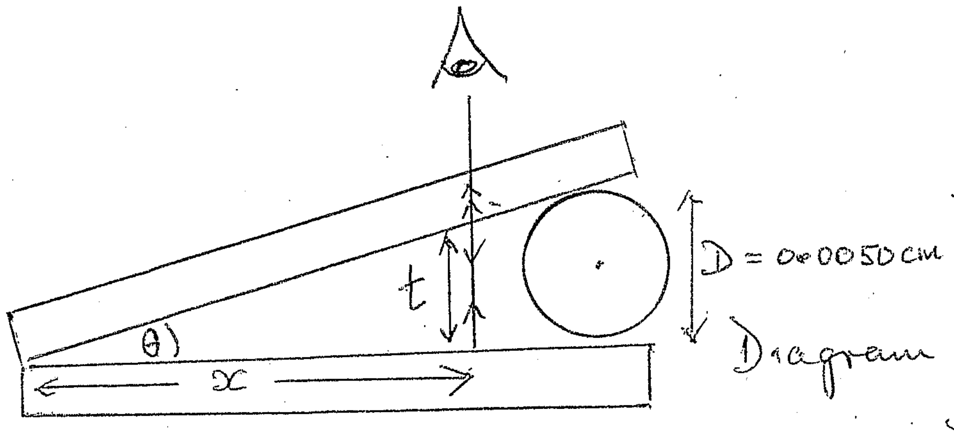
$f = \frac{1}{2\pi}\sqrt{\frac{g}{A}}$ 1

$= \frac{1}{2\pi}\sqrt{\frac{9.81}{0.015 \times 10^{-2}}}$ 1

$f = 41$ Hz. 1

6

Q5(a)




1

Let the thickness of the air wedge be t at a distance x from the line of contact of the two rectangular slides. Light incident normally.
The conditions for interference, taking into account the phase change of $\pi$ at the air-glass interface: 1

CONSTRUCTIVE INTERFERENCE $2t = n\lambda + \frac{1}{2}\lambda$ $n = 0, 1, 2 \ldots$ (1) 1
n is a positive integer, or zero,
$\lambda$ is the wavelength

DESTRUCTIVE INTERFERENCE $2t = n\lambda$ $n = 0, 1, 2 \ldots$ (2) 1

PROBLEM For changes of $\Delta n$ in n and $\Delta t$ in t with
$t = 5.0 \times 10^{-5}$ m, $\therefore \Delta n = 200$ $\Delta t = t$

$$2\Delta t = \Delta n \lambda$$ from (1) or (2) 1

ie $2 \times (5.0 \times 10^{-5}) = 200\lambda$

$$\lambda = 5.00 \times 10^{-7} \text{ m}$$ 1

6

(b) When $2t = (0)\lambda = 0$ there is destructive interference for all wavelengths. This occurs along ~~the~~ contact line of the two plates. Beyond this destructive interference region there will be constructive interference for $2t = \frac{1}{2}\lambda$, which varies with $\lambda$. White light will produce destructive interference for $t \sim 0$ followed by a coloured region in the region $2t \sim \frac{1}{2}\lambda$, due to the range of wavelengths. Subsequently there are bands of coloured fringes that become increasingly 2

Q5 (b) whiter; the destructive regions of certain wavelengths overlapping with the constructive regions of other wavelengths.

For a transparent liquid of refractive index $\mu$ the optical path is $2\mu t$

Thus:

Constructive interference occurs when $2\mu t = n\lambda + \frac{1}{2}$ 1

Destructive interference occurs when $2\mu t = n\lambda$ 1

$n = 0, 1, 2, \ldots$ 4

(c) The interference fringe system, for monochromatic light, gives contours of constant air gap thickness. For perfectly flat plates, in a wedge arrangement, these are straight lines parallel to the line of contact of the plates. The deviation of these contours shows the variation in ~~the air~~ gap thickness. From the value of $n$ one can determine the 'contour map' variation and its deviation from the perfectly flat plates interference pattern. 4

4

(d) For wavelengths $\lambda_1$ and $\lambda_2$ and air gap thicknesses $t_1$ and $t_2$, one obtains for constructive interference

$$2t_1 = n_1\lambda_1 + \tfrac{1}{2}\lambda_1$$

and

$$2t_2 = n_2\lambda_2 + \tfrac{1}{2}\lambda_2$$

If for a particular set of $n_1$ and $n_2$ the fringes coincide for common thickness $t$, Q5 (d)

$$2t = n_1\lambda_1 + \tfrac{1}{2}\lambda_1$$

and

$$2t = n_2\lambda_2 + \tfrac{1}{2}\lambda_2$$

Giving

$$\lambda_1 = \left(\frac{2n_2+1}{2n_1+1}\right)\lambda_2 \qquad (1)$$

So counting the fringes to obtain $n_1$ and $n_2$, starting from $n_1 = n_2 = 0$, one can determine $\lambda_1$ from a knowledge of $\lambda_2$

3

OR

Alternatively

If there are several such coincidences, from (1),

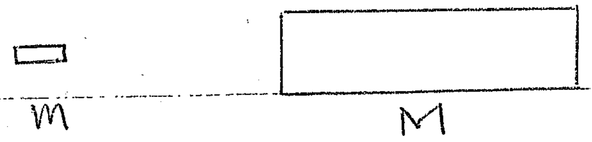
$$n_1 = n_2\left(\frac{\lambda_2}{\lambda_1}\right) + \frac{1}{2}\left(\frac{\lambda_2}{\lambda_1} - 1\right)$$

Plotting $n_1$ against $n_2$ gives a straight line with gradient $(\lambda_2/\lambda_1)$ and also from intercept $\frac{1}{2}(\lambda_2/\lambda_1 - 1)$. Hence obtaining $\lambda_1$ knowing $\lambda_2$.

3

6

Q6 (a)




Let initial speed of m be $u$ and final speed of M be $v$
Conservation of momentum requires

$$mu = Mv + m\left(\frac{u}{2}\right) \qquad 1$$

ie $$m\left(\frac{u}{2}\right) = Mv \qquad (1)$$

Energy lost given by

$$\Delta E = \frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{u}{2}\right)^2 - \frac{1}{2}Mv^2 \qquad 1$$
$$= \frac{3}{8}mu^2 - \frac{1}{2}Mv^2$$

Fraction of lost KE

$$\frac{\Delta E}{E} = \frac{\frac{3}{8}mu^2 - \frac{1}{2}Mv^2}{\frac{1}{2}mu^2} \qquad 1$$

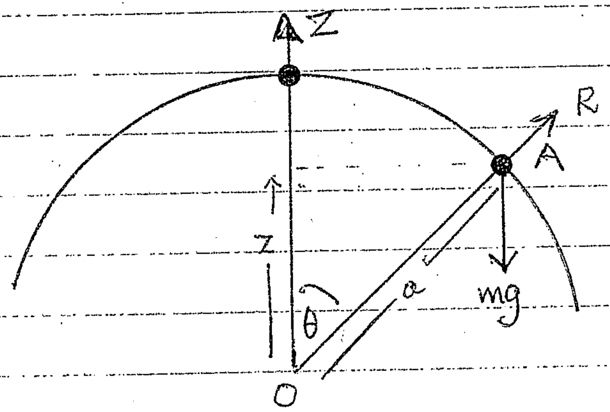
$$= \frac{\frac{3}{8}u^2 - \frac{1}{2}\frac{M}{m}v^2}{\frac{1}{2}u^2} \qquad 1$$

From (1) $$= \frac{\frac{3}{8}u^2 - \frac{1}{2}\left(\frac{M}{m}\right)\left(\frac{m}{M}\right)^2\left(\frac{u}{2}\right)^2}{\frac{1}{2}u^2}$$

$$\frac{\Delta E}{E} = \frac{3}{4} - \frac{1}{4}\left(\frac{m}{M}\right) \qquad 1$$

5

(b)




Let R be reaction of sphere on particle
It will leave the sphere when $R = 0$
Conservation of energy if at position $(z, \theta)$ with speed $v$; pe measured from O

$$\frac{1}{2}mv^2 + mgz = mga \qquad (1) \qquad 1$$

Equation of motion along AO

$$mg\cos\theta - R = \frac{mv^2}{a} \qquad 2$$

Q6 (b) Expressed in terms of z this becomes

$$mg\frac{z}{a} = \frac{mv^2}{a} + R \quad (2)$$ 1

From (1) and (2) eliminating $v^2$

$$mg\frac{z}{a} = \frac{m}{a}2g(a-z)+R$$
$$R = \frac{mg}{a}(3z-2a)$$ 1

It will leave surface of sphere when $R=0$ ie

$$\frac{mg}{a}(3z-2a)=0$$
$$\underline{z=\tfrac{2}{3}a}$$ 1

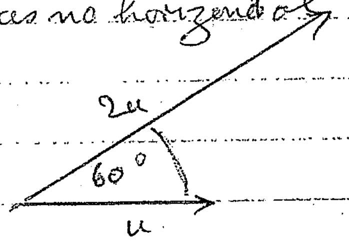
6

(c) Conservation of momentum requires W to move to LHS and w move to the RHS; so that their momenta cancel. 1

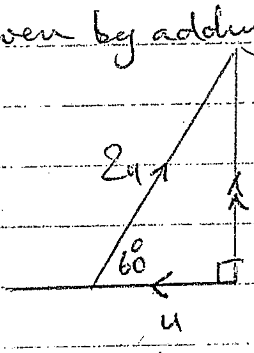
The only force acting on m is a vertical force. Thus m must move vertically downwards, having no horizontal component of velocity; it has no horizontal momentum 3

4

(d)




$\underline{2u}$ relative to $\underline{u}$ given by adding the vectors $\underline{2u}$ and $(-\underline{u})$ 1




This produces a right angled triangle with sides of lengths 2u, u and $\sqrt{3}u$ 1

2

So relative velocity is $\sqrt{3}u$ vertically in diagram 1

5

Q7 (a) For circular orbit at approx. Earth radius with velocity $v_E$, mass m,

$$\frac{mv_E^2}{R_E} = \frac{GM_E m}{R_E^2}$$ 1

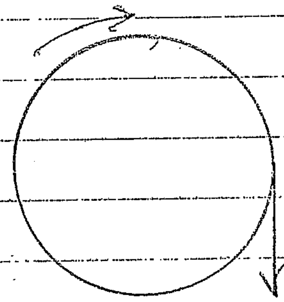
$$v^2 = \frac{M_E G}{R_E} = \frac{(5.97\times10^{24})(6.67\times10^{-11})}{6.83\times10^6}$$ 1

$$= 5.83\times10^7$$

$$v = 7.64\times10^3\ \text{ms}^{-1}$$ 1

3

(b) If Earth rotating clockwise, as in Fig, launching a space craft tangentially and clockwise from the equator will add $R_E\omega_E$ ($\omega_E$ is angular velocity of Earth) to the launch velocity. However, if launched from a diametrically opposite location on the Earth the launch velocity will be reduced by $R_E\omega_E$. Thus it is always an advantage to launch in the direction of rotation of the Earth. Intermediate results hold for other latitudes.



3

3

(c)

For circular motion in orbit, at distance R from centre of Earth, vehicle mass m:

$$\frac{mv^2}{R} = \frac{GM_E m}{R^2}$$

$$v^2 = \frac{GM_E}{R} \quad (1)$$ 1

Energy E: $E = \frac{1}{2}mv^2 - \frac{GM_E m}{R}$ 1

From (1) $= -\frac{1}{2}\frac{GM_E m}{R}$ from (1) 1

So if E reduces, E becomes larger in magnitude but negative, ie R becomes smaller.
So from (1) v increases.

3

7

Q7 (d) Assume the mass of the galaxy, $M_G$, is all concentrated at its centre. This is valid for a uniform distribution of mass in a sphere. It will be assumed that this is a good approximation. The star, mass $M_S$ and speed $V_S$, rotates in a circle around the centre, radius $R_G$ 2

Then 1

$$\frac{M_S V_S^2}{R_G} = \frac{G M_S M_G}{R_G^2} \qquad (1)$$

Now $R_G = (3\times10^4)(3\times10^8)(365\times24\times60\times60)$ 1

$= \underline{2.8\times10^{20}\text{ m}}$

From (1)

$$M_G = \frac{V_S^2 R_G}{G}$$

$$= \frac{(250\times10^3)^2(2.8\times10^{20})}{6.7\times10^{-11}}\text{ kg}$$

$$= \frac{10^{12}(2.8\times10^{20})}{16\,(6.7\times10^{-11})}\text{ kg}$$ 1

$M_G = \underline{2.6\times10^{41}\text{ kg}}$ * 1

(ii) $M_S = 2.0\times10^{30}$ kg equal to our Sun's mass

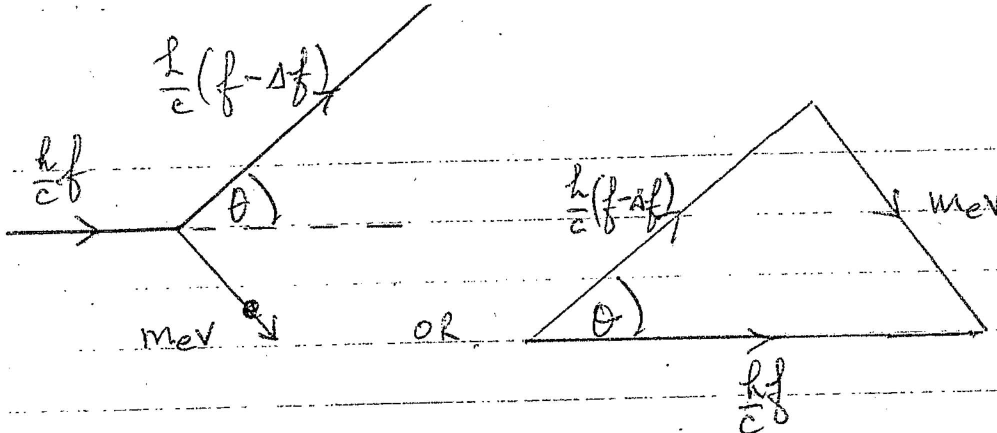
The number $N$ of stars is given by

$$N = \frac{M_G}{M_S} = \frac{2.6\times10^{41}}{2.0\times10^{30}} = \underline{1.3\times10^{11}} \; *$$ 1

* One sig. fig or just order of magnitude acceptable

7

Q8 (a)




3

OR BOTH DIAGRAMS, WITH LABELS

(b) $(m_e v)^2 = \left(\frac{h}{c}f\right)^2 + \left[\frac{h}{c}(f-\Delta f)\right]^2 - 2\left(\frac{h}{c}\right)^2 f(f-\Delta f)\cos\theta$ ① 4

(c) $hf = \frac{1}{2} m_e v^2 + h(f-\Delta f)$ ② 2

9

(d) ~~From~~ (2) $m_e v^2 = 2h\Delta f$ ③ 1

Substituting ③ into LHS of ①

$$2hm_e\Delta f = \left(\frac{h}{c}\right)^2 f^2 + \left(\frac{h}{c}\right)^2 (f-\Delta f)^2 - 2\left(\frac{h}{c}\right)^2 f(f-\Delta f)\cos\theta$$ 1

$$\Delta f = \frac{1}{hm_e}\left(\frac{h}{c}\right)^2\left\{[1-\cos\theta]f^2 + f\Delta f(1-\cos\theta) + \frac{1}{2}(\Delta f)^2\right\}$$ 1

$$\Delta f\left[1 - \frac{hf}{m_e c^2}(1-\cos\theta)\right] = \left(\frac{hf}{m_e c^2}\right) f\left[(1-\cos\theta) + \frac{1}{2}\left(\frac{\Delta f}{f}\right)^2\right]$$ 1

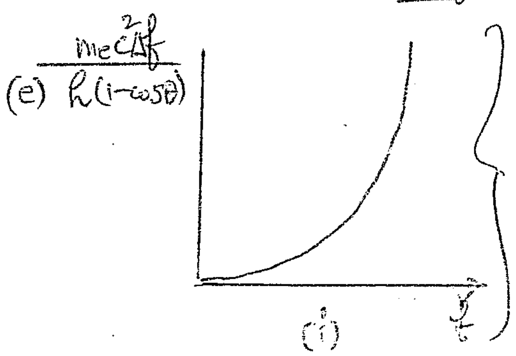
$$\Delta f = \left(\frac{hf}{m_e c^2}\right) f\left[(1-\cos\theta) + \frac{1}{2}\left(\frac{\Delta f}{f}\right)^2\right] \Big/ \left[1 - \frac{hf}{m_e c^2}(1-\cos\theta)\right]$$ 1

As $\frac{hf}{m_e c^2} \ll 1$ and $\left(\frac{\Delta f}{f}\right) \ll 1$, we obtain the approximation 1

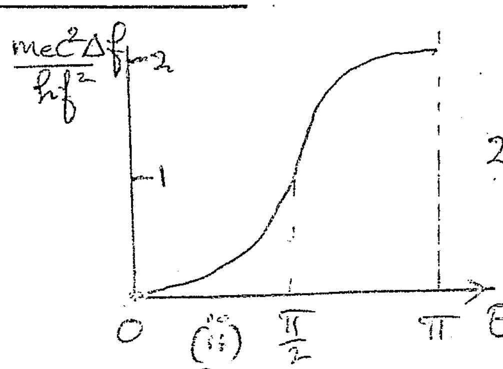
$$\Delta f = \frac{hf^2}{m_e c^2}(1-\cos\theta)$$ 1

7

(e)




1 Mark



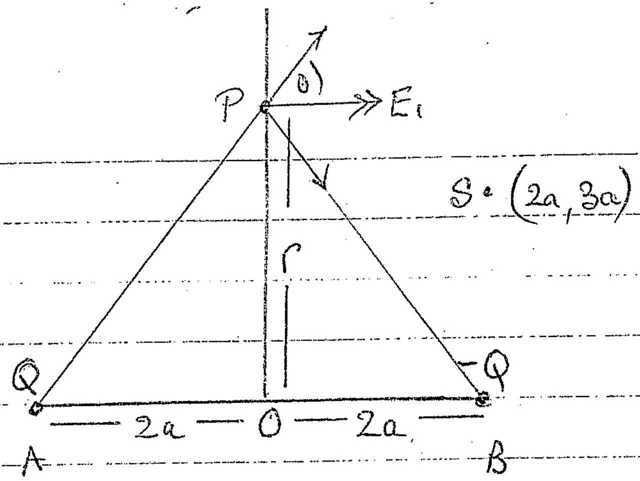

2 Marks

3

max. $\Delta f$ in (ii) gives $\Delta f = \frac{2hf^2}{m_e c^2}$ for $\theta = \pi$ 1

4

Q9 (a)




$$V_1 = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{r^2+(2a)^2}} - \frac{1}{\sqrt{r^2+(2a)^2}}\right] = 0$$ 1

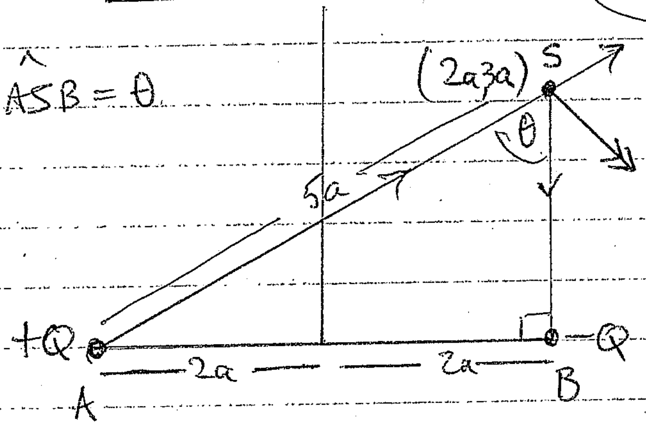
By symmetry $E_1$ is parallel ~~to~~ $\overline{AB}$ at P. Let $E_1$ make an angle $\theta$ with AP and BP. Resolving the forces in this direction

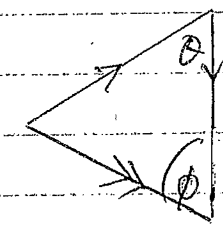
$$E_1 = \frac{2Q}{4\pi\varepsilon_0}\left(\frac{1}{r^2+(2a)^2}\right)\cos\theta$$ 1

$$= \frac{2Q}{4\pi\varepsilon_0}\left(\frac{1}{r^2+(2a)^2}\right)\frac{2a}{\sqrt{r^2+(2a)^2}}$$ 1

$$= \frac{Q}{\pi\varepsilon_0}\frac{a}{(r^2+(2a)^2)^{3/2}}$$ parallel to AB at P $\frac{2}{5}$

(b) Let $\hat{ASB} = \theta$




TRIANGLE OF FORCES

$$V_2 = \frac{Q}{4\pi\varepsilon_0}\left[\frac{-1}{3a} + \frac{1}{5a}\right]$$

$$V_2 = \frac{-Q}{4\pi\varepsilon_0 a}\left(\frac{2}{15}\right)$$ 1

Resultant field $E_2$ given by

$$E_2^2 = \left(\frac{Q}{4\pi\varepsilon_0}\right)^2\left[\frac{1}{(3a)^4} + \frac{1}{(5a)^4} - 2\left(\frac{1}{3a}\right)^2\left(\frac{1}{5a}\right)^2\cos\theta\right]$$ 2

$$= \left(\frac{Q}{4\pi\varepsilon_0 a^2}\right)^2\left[\frac{1}{3^4} + \frac{1}{5^4} - 2\left(\frac{1}{3^2}\right)\left(\frac{1}{5^2}\right)\left(\frac{3}{5}\right)\right]$$ as $\cos\theta = \frac{3}{5}$ 1

Q9

$$E_2^2 = \left(\frac{Q}{4\pi\varepsilon_0 a^2}\right)^2 \frac{1}{3^4 5^4}\left[5^4+3^4-2(3^2)(5^2)\left(\frac{3}{5}\right)\right]$$ 1

$$= \left(\frac{Q}{4\pi\varepsilon_0 a^2}\right)^2 \left(\frac{1}{3^4}\right)\left(\frac{1}{5^4}\right)\left[706-270\right]$$

$$E_2 = \left(\frac{Q}{4\pi\varepsilon_0 a^2}\right) \frac{\sqrt{436}}{225}$$ 1

$$\underline{E_2 = \left(\frac{Q}{4\pi\varepsilon_0 a^2}\right) 0.0928}$$ 1

Direction of $\bar{E}_2$ angle $\phi$ to vertical as in diagram

Using sin rule:

$$\frac{E_2}{\sin\theta} = \frac{\frac{Q}{4\pi\varepsilon_0 a^2(5^2)}}{\sin\phi}$$ 1

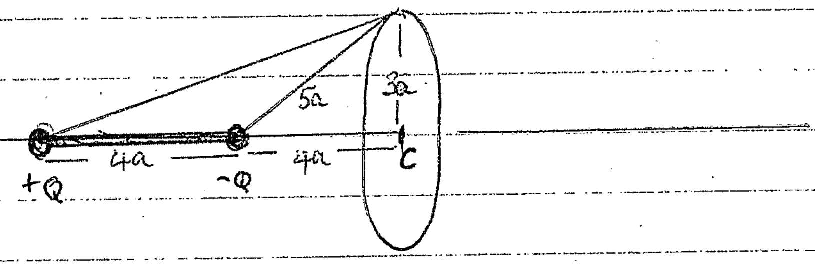
$$\frac{0.0928}{4/5} = \frac{0.040}{\sin\phi}$$

$$\sin\phi = 0.040\left(\frac{4}{5}\right)\Big/(0.0928) = 0.345$$ 1

$$\underline{\therefore \phi = 20.2^\circ}$$ 1

10

(c)




Initial potential $V_1$

$$V_1 = \frac{Q^2}{4\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{8^2+3^2}} - \frac{1}{5}\right)$$ 1

$$= -\frac{Q^2}{4\pi\varepsilon_0 a}(0.0830)$$ $\frac{1}{2}$

Final potential $V_2$

$$V_2 = \frac{Q^2}{4\pi\varepsilon_0 a}\left(-\frac{1}{3}+\frac{1}{5}\right)$$ 1

$$= -\frac{Q^2}{4\pi\varepsilon_0 a}(0.133)$$ $\frac{1}{2}$

$$\frac{1}{2}mv^2 = V_1 - V_2 = \frac{Q^2}{4\pi\varepsilon_0 a}(0.0504)$$ 1

$$v = \frac{\sqrt{2(0.0504)}\,Q}{\sqrt{4\pi\varepsilon_0 a m}} = \frac{Q}{\sqrt{2\pi\varepsilon_0 a m}}(0.224) = \frac{0.224Q\sqrt{2\pi\varepsilon_0 a m}}{2\pi\varepsilon_0 a m}$$ 1

5