2012 - 2013 ROUND 1

19/11/12

2013 BPhO PAPER 1

MARK

SOLUTIONS

Q1

(a) Reject 3·66 as it deviates markedly from the other result – possibly due to a misprint — 1

Average of remaining 10 measurements = 6·65 — 1

(i) Best estimate 6·65

(ii) Accuracy based on standard deviation or average of modulus of deviation from mean or any other method

Accept accuracy in range $\pm(0.05 \to 0.03)$ — 1

or standard error of the mean, which is $\pm(0.0149)$

3

(b)

(i) Upwards — 1

(ii) Rope B moves down with velocity $\frac{b}{a}V$ — 1

Rope D moves up with velocity V — 1

Centre of P and W rise with speed $\frac{1}{2}V\left(1-\frac{b}{a}\right)$ — 1

$= V\left(\frac{a-b}{2a}\right)$ — 1

5

(c) At the surface of the Earth, for mass m,

$$mg = \frac{GM_E m}{R_E^2} \qquad M_E \text{ mass of Earth}$$

$$g = \frac{GM_E}{R_E^2}$$ — 1

If ρ density of Earth, assumed homogeneous, — 1

$$g = \frac{G\,\frac{4}{3}\pi R_E^3 \rho}{R_E^2}$$ — 1

$$\rho = \frac{g}{\frac{4}{3}\pi G R_E}$$

$$= \frac{9.81}{1.33\pi(6.67\times10^{-11})(6.38\times10^{6})} \ \text{kg m}^{-3}$$ — 1

$\rho = 5.51 \ \text{g cm}^{-3}$ — 1

5

Q1 (d) Activity $\quad A(t) = \frac{dN}{dt} = -\lambda N_0 e^{-\lambda t} \quad (N = N_0 e^{-\lambda t})$ 1

At $t=0$ $\quad A(0) = \left(\frac{dN}{dt}\right)_0 = -\lambda N_0$

After 10^8 s, $\quad A(10^8) = -\lambda N_0 e^{-\lambda 10^8}$ 1

$$\frac{A(10^8)}{A(0)} = \frac{\lambda N_0 e^{-\lambda 10^8}}{\lambda N_0} = (1 - 0.010) = 0.990$$ 1

Thus

$$\lambda 10^8 = -\ln(0.990)$$

As $T = \ln 2/\lambda$,

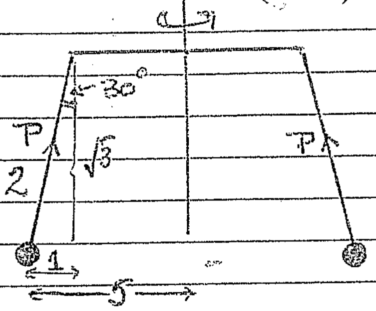
$$T = -\frac{10^8 \ln 2}{\ln(0.990)}$$ 1

$$\underline{T = 6.9 \times 10^9 \text{ s}} \quad (\text{or } 219 \text{ years})$$ 1

5

(e)

Let v be the velocity of the spheres and P the tension in the rope. m is mass of spheres.



For circular motion, resolving horizontally,

$$\frac{mv^2}{5.0} = \frac{1}{2}P \quad (1)$$ 2

Resolving vertically

$$mg = P\cos 30^\circ = P\frac{\sqrt{3}}{2} \quad (2)$$ 1

Substituting (2) into (1) $\quad v^2 = \frac{5g}{\sqrt{3}} \quad (3)$

Period $\quad T = \frac{2\pi(5)}{v} = 10\pi\sqrt{\frac{\sqrt{3}}{5g}}$ from (3) 1

$$\underline{T = 5.09 \text{ s}}$$ 1

5

(f) Using usual notation

$$hf = W + Ve$$ 2

$$V = \frac{hf - W}{e}$$

$$= \frac{(6.3 \times 10^{14})(6.63 \times 10^{-34})}{1.60 \times 10^{-19}} - \frac{3.7 \times 10^{-19}}{1.60 \times 10^{-19}}$$ 1

$$= \frac{4.769 \times 10^{-20}}{1.60 \times 10^{-19}}$$

$$V = 0.298 \text{ V}$$ 1

4

$$\simeq 0.30 \text{ V} \quad (\text{to 2 sig. fig})$$

Q1 (g) Rate of heating, $12 = s(\text{mass flowing}/s)(\text{rise in temperature})$ 1

$$= s\left(\frac{0.060}{60}\right)(2.0)$$

$$s = 6.0 \times 10^{3}\ \text{J kg}^{-1}$$ 1

2

(h) Using standard notation

$$E_1 = \tfrac{1}{2} C_1 V_1^2 \qquad \text{where } V_1 = 10^5\ \text{V}$$ 1

and $C_1 = \frac{\varepsilon A_1}{d_1}$ A_1 area of plates, d_1 separation of plates 1

Thus

$$E_1 = \frac{1}{2}\frac{(8.85\times10^{-12})(25\times10^{6})}{750}(10^5)^2$$

$$E_1 = 1475\ \text{J} \qquad (\text{Accept } 1.47\times10^{3} \text{ to } 1.48\times10^{3})$$ 1

(i) Energy increases as work has to be done to further separate charge on cloud against attraction from opposite charge on the ground; charge on cloud constant. 1

(ii) $E = \frac{1}{2} C_2 V_2^2$ C_2 is the new capacitance, V_2 new potential 1

$$= \frac{1}{2} C_2 \left(\frac{Q_1}{C_2}\right)^2 \quad \text{as } V_2 = \frac{Q_1}{C_2}$$

where Q_1 is the charge on the cloud that remains constant.

Now $C_2 = \frac{\varepsilon A_1}{d_2}$ and $Q_1 = C_1 V_1 = \frac{\varepsilon A_1 V_1}{d_1}$

B

So

$$E_2 = \frac{1}{2}\frac{Q_1^2}{C_2}$$

$$= \frac{1}{2}\left(\frac{\varepsilon A_1 V_1}{d_1}\right)^2 \frac{d_2}{\varepsilon A_1}$$

$$= \frac{1}{2}(8.85\times10^{-12})(25\times10^{6})(10^5)^2 \frac{1250}{(750)^2}$$

$$= E_1 \frac{1250}{750} = 2458\ \text{J}$$ 1

Increase in energy $\Delta E = 2458 - 1475$

$$\Delta E = 983\ \text{J}$$ 1

10

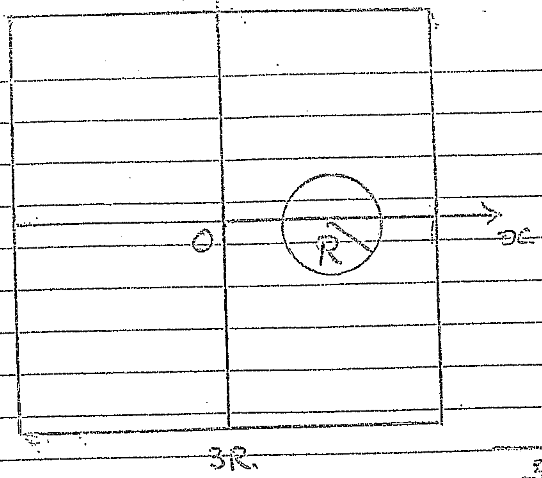
Alternative methods acceptable

Q1 (i)

(i) By symmetry C. of G. along x-axis
Moments about y-axis gives $\bar{x}$:

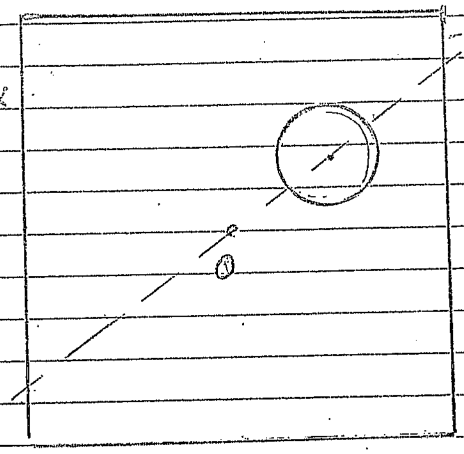
$$(9R^2 - \pi R^2)\bar{x} = -x\pi R^2$$ 1

$$\bar{x} = -\frac{\pi x}{(9-\pi)}$$ 1



3

(ii) METHOD I
Let line through (x, y) be $y = \alpha x$
By symmetry C. of G. along line through O and (x, y)
Measuring distances along line $y = x\alpha$ 1
Moments: $(s = \sqrt{x^2 + y^2})$



$$(9R^2 - \pi R^2)\bar{s} = -s\pi R^2$$

$$\bar{s} = \frac{-s\pi R^2}{9R^2 - \pi R^2}$$ 1

$$= \frac{-\pi\sqrt{x^2+y^2}}{9-\pi}$$ 1

3

OR ALTERNATIVE METHOD

x-coord. of C. of G., $\bar{x}$, is given by taking moments about y-axis:

$$(9R^2 - \pi R^2)\bar{x} = -x\pi R^2$$ 1

$$\bar{x} = -\frac{x\pi}{9-\pi}$$ 1

Similarly

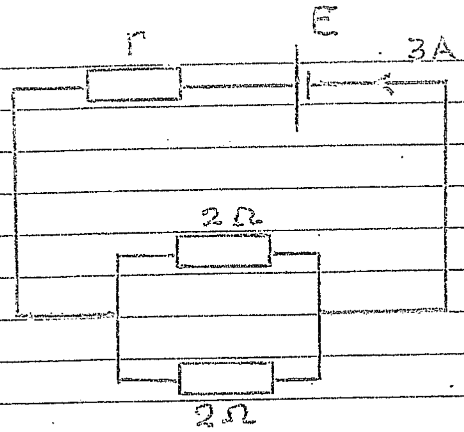
$$\bar{y} = -\frac{y\pi}{9-\pi}$$ 1

3

Coords of C. of G. $(\bar{x}, \bar{y})$

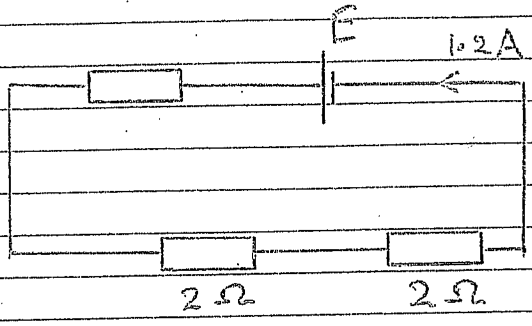
TOTAL 6

Q1 (g)



$E = 3\left[r + \left(\frac{1}{2}+\frac{1}{2}\right)^{-1}\right]$

$E = 3\,[r+1]$ (1) 1



$E = 1.2\,(r+4)$ (2) 1

From (1) and (2) $3(r+1) = 1.2\,(r+4)$

$r = 1\ \Omega$ $\frac{1}{2}$

Sub.

$E = 6\ V$ $\frac{1}{2}$

1

Power dissipated in each 2Ω resistor in series circuit

$W = 2\,(1.2)^2 = \underline{2.88\ W}$ 1

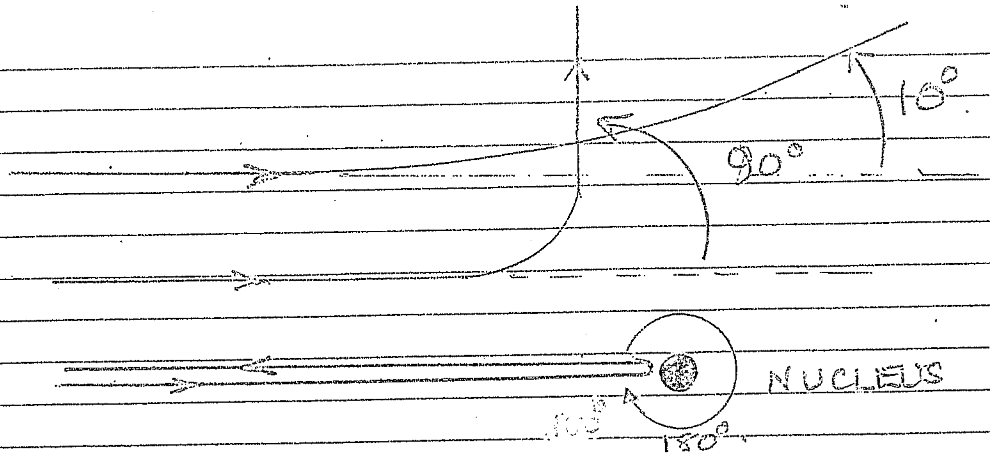
Power dissipated in each 2Ω resistor in parallel circuit

$W = \left(\frac{3}{2}\right)^2 2$

$\underline{W = 4.5\,W}$ 1

5

Q1 (k)



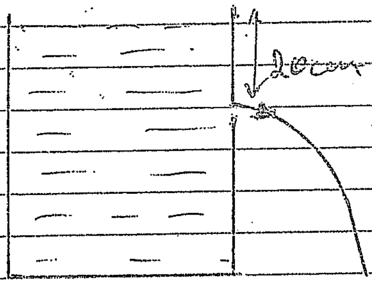
$\frac{1}{2}$ mark for each approx. deflection angle correct

$\frac{1}{2}$ mark is 180° deflection closest to nucleus, 10° deflection furthest and 90° intermediate } 3

At closest approach relative velocity is zero, relative to nucleus particle goes from approaching nucleus to receding from nucleus } 2

5

(l)



$\frac{1}{2}mv^2 = mgh$ for element of water mass m 1

$v^2 = 2gh$

$v = \sqrt{2(9.81)(0.20)}$

$v = 1.98\ ms^{-1}$ 1

Horizontal velocity of emerging water $1.98\ ms^{-1}$

Time, t, to reach the floor is given by "$s = ut + \frac{1}{2}gt^2$"

giving,

$0.80 = \frac{1}{2}(9.81)t^2$ 1

$t = \sqrt{\frac{1.60}{9.81}}$

$t = 0.403\ s$ 1

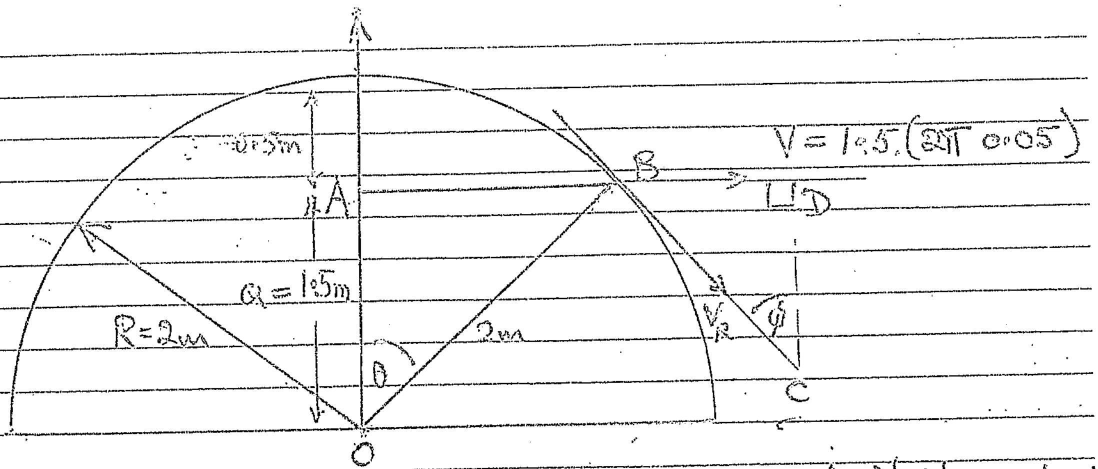
Horizontal distance travelled

$H = (1.98)(0.403)$

$H \approx 80\ cm$ 1

5

Q1 (iii)



As seen by observer puck moves with constant velocity '$V = r\omega$' 1

$$V = 1.5\,(2\pi\; 0.05)$$
$$= 0.471\ ms^{-1}$$ 1

(i) Time on turntable, t) given by

$$Vt = \sqrt{R^2 - a^2}$$ 1
$$= \sqrt{2^2 - (1.5)^2}$$

Giving

$$t = 2.8\,s$$ 1

(ii) $V = 1.5\,(2\pi\; 0.05)$ and rim speed of turntable $V_R = 2.0(2\pi\, 0.05)$ 1

Now velocity Triangle BCD is similar to OAB as velocities proportional to lengths OA and OB, $B\hat{D}C = 90°$. 1

To obtain velocity of puck leaving turntable relative to student, reverse $\underline{V}_R$ and add vectorally to V. 1

Hence relative velocity of puck is, where ϕ is angle to tangent,

$$\sqrt{V_R^2 - V^2} \quad \text{at angle } \phi = \sin^{-1}\left(\frac{1.5}{2.0}\right) \text{ to tangent}$$ 1

ie $2\pi(0.05)\sqrt{1.75}$ at angle $\phi = 48.6°$ to tangent

ie $0.416\ ms^{-1}$ at angle $\phi = 48.6°$ to tangent 2

10

Q1 (m) Area of loop $= (0.04)^2 = 1.6 \times 10^{-3}\ m^2$

If ϕ is the flux due to the changing field the pd V is given by

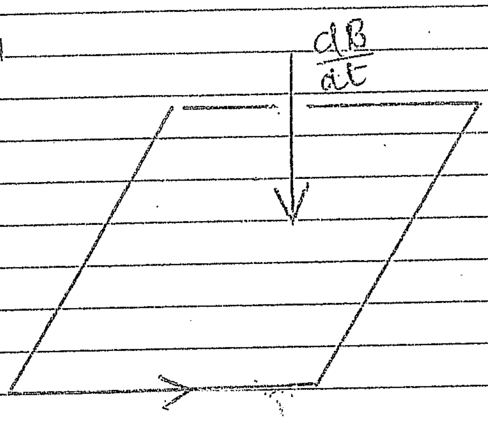
$$V = \frac{d\phi}{dt} = 1.6 \times 10^{-3}(0.30) = 0.48 \times 10^{-3}\ V$$ 2

As $R = 2 \times 10^{-3}\ \Omega$, current I given by

$$I = \frac{0.48 \times 10^{-3}}{2.00 \times 10^{-3}}$$

$$I = 0.24 A$$ 1

DIAGRAM



Correct Diagram 1

The current in the loop must produce a magnetic field that opposes increasing (dB/dt), Lenz's Law

4

(c) Let h be height reached

(i) Energy $E = 60 \times 12 \times 60 \times 60$. J

$= 12(60^3)$ 1

$= mgh$

Thus $20(9.81)h = 12(60^3)$ 1

$h = 1.32 \times 10^4\ m$ 1

(ii) $1(9.81)h = 4 \times 10^7$ 1

$h = 4.08 \times 10^6\ m$ 1

5

Q1 (p) If ρ is the density of the soap film, radius R, ΔR its thickness then

mass of bubble $M_B = 4\pi R^2 \Delta R \rho$ $\frac{1}{2}$

Upthrust, U, due to surrounding air is given by

$$U = \frac{4}{3}\pi R^3 (1.28 - 0.18) g$$ $\frac{1}{2}$

Now $M_B g = U$

Thus

$$4\pi R^2 \Delta R \rho g = \frac{4}{3}\pi R^3 (1.28 - 0.18) g$$ 1

Giving

$$\Delta R = \frac{(1.5\times10^{-2})(1.10)}{3\times10^3}.$$

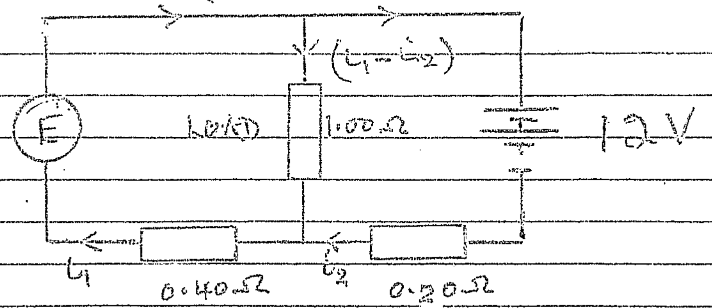
Dividing by $\lambda_{red} = 650\times10^{-9}$ m

$$\frac{\Delta R}{\lambda_{red}} = \frac{(1.10)(1.5\times10^{-2})}{(3\times10^3)(650\times10^{-9})}$$ 1

$$\frac{\Delta R}{\lambda_{red}} = 8.46$$ 1

4

Q2 (i)



This occurs when $i_2 = 0$

LH loop gives

$$E = (1.00 + 0.40)\,i_1$$
$$= 1.40\, i_1 \quad (1)$$ 1

RH loop gives

$$12 = 1.00\, i_1 \quad (2)$$ 1

Subst for i_1 from (2) into (1)

$$E = 16.8\ \text{V}$$ 1

(ii) Power efficiency $= \dfrac{i_1^2\,(1.00)}{i_1^2\,(1.40)} \times 100\ \%$

$$= 71\%$$ 1

(iii) LH loop:

$$20 = 1.00\,(i_1 - i_2) + 0.40\, i_1$$
$$20 = 1.40\, i_1 - i_2 \quad (3)$$ 1

RH loop:

$$12 = 1.00(i_1 - i_2) - 0.20\, i_2$$
$$12 = i_1 - 1.2\, i_2 \quad (4)$$ 1

Substituting i_1 from (4) into (3)

$$20 = 1.4(12 + 1.2\, i_2) - i_2$$
$$= 16.8 + 0.68\, i_2$$ 1
$$i_2 = \frac{3.2}{0.68} = 4.7\ \text{A}$$ 1

8

(i) (b) By symmetry, reversing V gives the same arrangement. So 1

$i_3 = i_4$ 1

$i_1 = i_2$ 1

$i_6 = 0$

(ii)

As $i_6 = 0$ circuit reduces to 3 resistors in parallel

$$\therefore R_{eq} = \left(\tfrac{1}{2} + \tfrac{1}{2} + \tfrac{1}{4}\right)^{-1}$$ 1

$$R_{eq} = \frac{4}{5}\ \Omega \quad (0.8\,\Omega)$$ 1

10

Q2 (b) (iii)

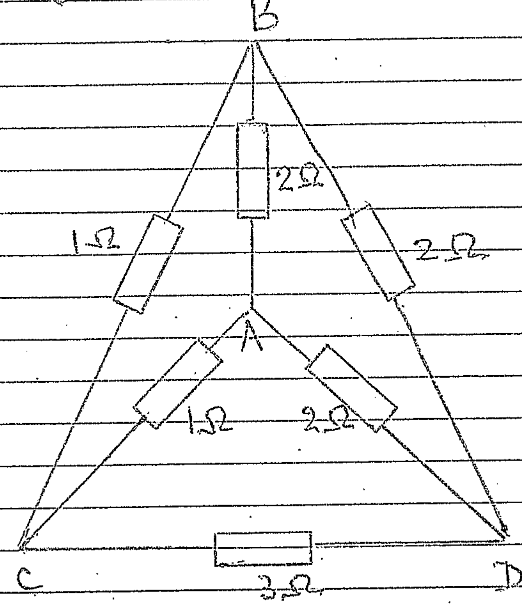
$i_5 = \frac{1}{5}V$

$i_3 = i_4 = \frac{1}{2}V$

$i_1 = i_2 = \frac{1}{4}V$ 1

$i_6 = 0$

(iv)



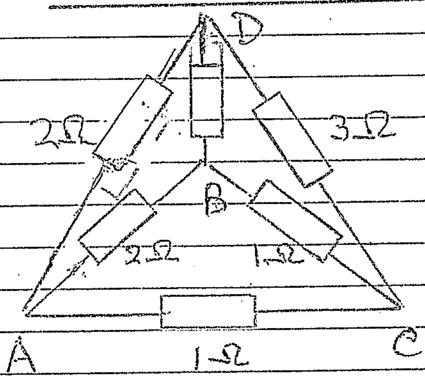
Drawing 1

The circuit is a balanced Wheatstone bridge, so $i_{AB} = 0$ 1

Resistance across consists 3 3Ω resistance in parallel

$$R_{CD} = \left(\frac{1}{3} + \frac{1}{3} + \frac{1}{3}\right)^{-1} = 1\ \Omega.$$ 1

(v)



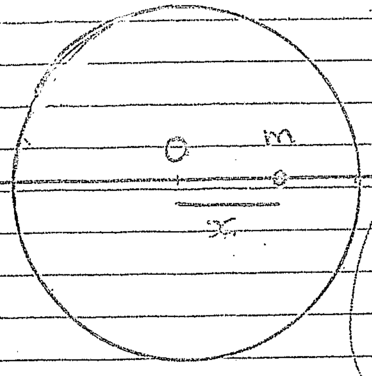
Drawing 1

This circuit cannot be reduced to a Wheatstone bridge balanced circuit. Consequently the resistance across AC cannot be expressed as a combination of resistances in series and parallel. 2

12

Q3 (a) Equation of motion

$$m\ddot{x} = \frac{Gm\left(\frac{4}{3}\pi x^3 \rho\right)}{x^2}$$



as attraction due to sphere radius x.

Now mass of the Earth M_E given by

$$M_E = \frac{4}{3}\pi R_E^3 \rho$$

Thus

$$\frac{4}{3}\pi x^3 \rho = \frac{x^3}{R_E^3} M_E$$

Giving

$$m\ddot{x} = -\frac{Gm\,x^3 M_E}{x^2 R_E^3}$$

$$\ddot{x} = -\frac{GM_E}{R_E^3}\,x$$ 3

This is SHM.

The period T_1 is given by

$$T_1 = 2\pi\sqrt{\frac{R_E^3}{GM_E}} \quad (1)$$ 1

However at the surface of ~~the~~ Earth

$$mg = \frac{GM_E m}{R_E^2}$$ 1

Giving

$$g = \frac{GM_E}{R_E^2} \quad (2)$$

Substituting for M_E from (2) into (1)

$$T_1 = 2\pi\sqrt{\frac{R_E}{g}}$$ 1

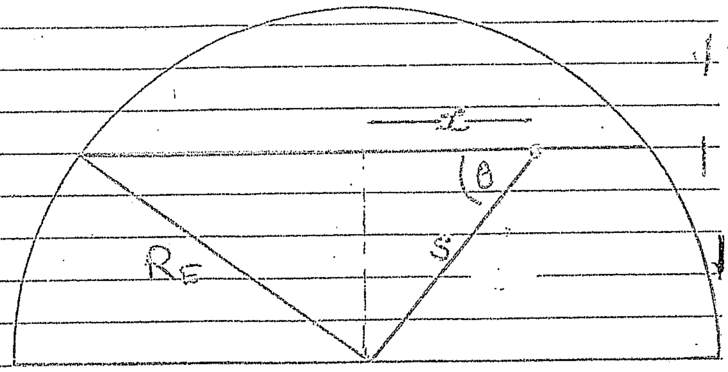
$$= 2\pi\sqrt{\frac{6.38\times10^6}{9.81}}$$ 1

$$T_1 = 5.07\times10^3\ \text{s}$$ 1

≈ 1 hour, 24 mins, 27 sec

8

Q3 (b) Equation of motion



$$m\ddot{x} = -\frac{Gm}{S^2}\left(\frac{S^3}{R_E^3}M_E\right)\cos\theta$$ 1

$$= -\frac{GmM_E S\cos\theta}{R_E^3}$$ 1

$$= -\frac{GmM_E x}{R_E^3}$$ 1

$$\ddot{x} = -\frac{GM_E}{R_E^3}x$$ 1

This is SHM with same period as in (a) so $T_1 = T_2 = 2\pi\sqrt{\frac{R_E}{g}}$ 1

5

(c) For circular motion in close Earth orbit with velocity v

$$\frac{mv^2}{R_E} = \frac{GM_E m}{R_E^2}$$ 1

$$v = \sqrt{\frac{GM_E}{R_E}} \quad (1)$$ 1

Period of motion $T_S = \frac{2\pi R_E}{v} = 2\pi R_E\sqrt{\frac{R_E}{GM_E}}$ from (1)

Now $g = \frac{GM_E}{R_E^2}$, so

$$T_S = 2\pi\sqrt{\frac{R_E}{g}}$$ 2

Thus $T_S = T_1 = T_2$ 1

5

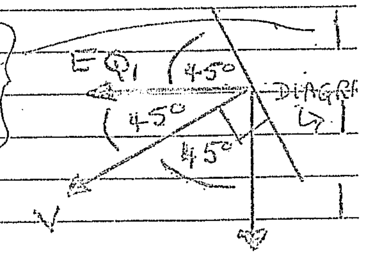
(d) By Kepler's first law the trajectory will be an ellipse with the centre of the Earth as the focus. 2

20

Q4 In air

(a) ~~In a~~ vacuum the drop falls with under the action of the gravitation force and the viscous force, proportional to the velocity, retarding the motion. The speed of the drop increases from rest until 'mg' and the viscous force balance when the drop falls with constant velocity. 2

(b) Viscous force in opposite direct to velocity. Resolving force perpendicular to this direction, with electric field E and charge Q_1



$$mg\cos 45^\circ = EQ_1 \sin 45^\circ$$

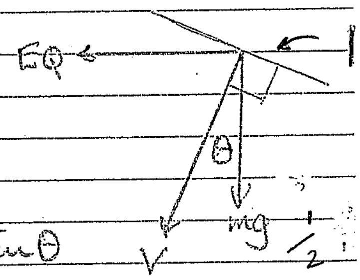
$$Q_1 = \frac{mg}{E}$$

$$= (3.3\times10^{-15})(9.81)\left(\frac{3.0\times10^{-2}}{2.0\times10^{3}}\right)$$

$$Q_1 = 4.95\times10^{-19}\,C$$

5

(c) Drop aquires additional charges
Let charge be Q travelling at angle θ to vertical
Resolving forces perpendicular to velocity



$$mg\sin\theta = EQ\cos\theta$$

$$Q = \frac{mg\tan\theta}{E}$$

$$Q = (3.3\times10^{-15})(9.81)(1.5\times10^{-5})\tan\theta$$ ½

$$Q = 4.86\times10^{-19}\tan\theta \quad (1)$$

$\theta = 18.43^\circ$ for charge Q_2

Sub^g into (1), $Q_2 = 4.86\times10^{-19}(0.3332)$

$$Q_2 = 1.62\times10^{-19}\,C$$

$\theta = 33.70^\circ$ for charge Q_3

Sub^g. into (1)

$$Q_3 = 3.24\times10^{-19}\,C$$ ½

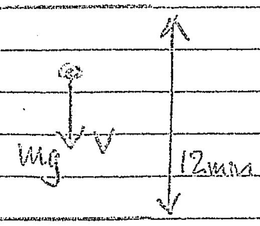
Best estimate is the lowest common denominator

namely $1.62\times10^{-19}\,C$

6

Q14

(d) $mg = kv$ 1



Substituting

$$10^{-14}(9.81) = k\,(4.0\times10^{-4})$$

$$\underline{k = 2.65\times10^{-10}}$$ 1

Now for n charges

$$\frac{V}{d}(ne) = mg + kv$$ 2 1

Substituting numerical values,

$$n\,\frac{1.5\times10^{3}\,(1.60\times10^{-19})}{12\times10^{-3}} = 10^{-14}(9.81) + 2.65\times10^{-10}(8\times10^{-5})$$

$$= 11.9\times10^{-14}$$ 2

$$n = \frac{10^{-14}\,(11.9)(12)}{10^{-13}\,(1.5)(1.60)}$$

$$\underline{n = 6}$$ 1

7

Q5

(i) For constructive interference of order p for wavelength λ,

$$2\mu t = p\lambda + \tfrac{1}{2}\lambda \quad (1)$$ 2

(reflection at front face of film gives rise to an additional phase change associated with a path length of $\tfrac{1}{2}\lambda$.

For the violet light (1) gives for $p=0$ 1

$$2(1.45)\,t_v = \tfrac{1}{2}\lambda$$

$$2.90\,t_v = \tfrac{1}{2}(420)10^{-9}$$

$$t_v = 72.4\text{ nm}$$ 1

4

(ii) As t_v occurs 3.0 cm from film top

$$x_v\phi = t_v$$ 1

$$\phi = \frac{72.4\times10^{-9}}{3.00\times10^{-2}}$$ 1

$$\phi = 2.41\times10^{-6} \quad \text{radians}$$ 1

OR $= 1.38\times10^{-4}$ degrees 3

(iii) For red light

$$2nt_R = \tfrac{1}{2}\lambda$$ 1

Distance

$X_R\phi = t_R$ $\quad (2.90)\,t_R = \tfrac{1}{2}(680\times10^{-9})$

$X_R = \dfrac{117\times10^{-9}\text{m}}{2.41\times10^{-6}}$ $\quad t_R = 117\text{ nm}$ 1

$= 4.85\text{cm}$ (1 mark) 3

(iv) $t_v = \phi\, x_v$ (1) 1

t_v remains constant as ϕ and x_v change

$$t_v = (\phi+\Delta\phi)(x_v+\Delta x_v)$$ 1

$$= \phi x_v + \Delta\phi\, x_v + \Delta x_v\phi + \Delta\phi\Delta x_v \quad (2)$$ 1

Subtracting (1) from (2) and neglecting $\Delta\phi\Delta x_v$ 1

$$0 = \Delta\phi\, x_v + \Delta x_v\,\phi$$ 1

Dividing by Δt $\quad 0 = \dfrac{\Delta\phi}{\Delta t}x_v + \dfrac{\Delta x_v}{\Delta t}\phi$ 1

Q5

(iv) Now $\frac{\Delta\phi}{\Delta t} = 8.3\times10^{-5}$ degrees per minute

So velocity of violet fringe, $\frac{\Delta x_v}{\Delta t}$, is given by

$$\frac{\Delta x_v}{\Delta t} = -\frac{1}{\phi}\frac{\Delta\phi}{\Delta t}x_v$$ 1

~~Now $-\frac{\Delta\phi}{\Delta t} = 8.3\times10^{-5}$ degrees per minute~~

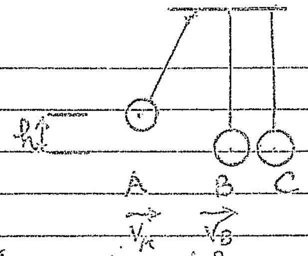
Sub^g from (1), $\frac{\Delta x_v}{\Delta t} = \frac{8.3\times10^{-5}}{1.38\times10^{-4}}(3.00\times10^{-2})$ 1

Velocity of fringe $= 1.8\times10^{-2}$ m/min

$V_v = 1.8$ cms/min 1

10

Q6



First Collision between A and B

Let V_A and V_B be velocities of A and B after collision

Conservation of momentum before and after A hits B with velocity $\sqrt{2gh}$

$$M\sqrt{2gh} = M(V_A+V_B)$$ 1

Conservation of energy

$$\tfrac{1}{2}M(2gh) = \tfrac{1}{2}M(V_A^2+V_B^2)$$ 1

Simplifying the equations

$$\sqrt{2gh} = V_A+V_B \quad (1)$$

$$2gh = V_A^2+V_B^2 \quad (2)$$

$$= (\sqrt{2gh}-V_B)^2+V_B^2 \quad \text{from } (1)$$

$$= 2gh - 2V_B\sqrt{2gh} + V_B^2+V_B^2$$

$$V_B(2\sqrt{2gh}) = 2V_B^2$$ 1

As $V_B \neq 0$, $\underline{V_B = \sqrt{2gh}}$ 1

Substituting into (1) $\underline{V_A = 0}$ 1

A is at rest and B has velocity $\sqrt{2gh}$ after the collision

Collision of B with C

Collision of B with C has same conservation equations as A with B as B initially has velocity $\sqrt{2gh}$.
Thus after collision B at rest and C has velocity $\sqrt{2gh}$ and consequently C rises to height h.
Now C starts at height h, with A and B at rest, and so the collisions are repeated, in reverse order, with the cycle repeated indefinitely. 3

8

Q6

(ii) Collision between A and B

Conservation of momentum (A and B have velocities V_A and V_B after collision)

$$M\sqrt{2gh} = MV_A + 2MV_B$$ 1

Conservation of energy

$$\tfrac{1}{2}M(2gh) = \tfrac{1}{2}MV_A^2 + \tfrac{1}{2}(2M)V_B^2$$ 1

Simplifying,

$$\sqrt{2gh} = V_A + 2V_B \quad (3)$$

$$2gh = V_A^2 + 2V_B^2 \quad (4)$$

Substituting V_A from (3) into (4)

$$2gh = (\sqrt{2gh} - 2V_B)^2 + 2V_B^2$$

Giving

$$2gh = 2gh - 4V_B\sqrt{2gh} + 4V_B^2 + 2V_B^2$$ 1

As $V_B \neq 0$,

$$4\sqrt{2gh} = 6V_B$$

$$\underline{V_B = \tfrac{2}{3}\sqrt{2gh}}$$ 1

Substituting into (3),

$$V_A = \sqrt{2gh} - 2V_B$$

$$= \sqrt{2gh} - 2\left(\tfrac{2}{3}\right)\sqrt{2gh}$$

$$= \sqrt{2gh}\left(1 - \tfrac{4}{3}\right)$$

$$\underline{V_A = -\tfrac{1}{3}\sqrt{2gh}}$$ 1

Collision of B with C (B and C have velocities V_B' and V_C' after collision)

Momentum conservation $\quad 2M\left(\tfrac{2}{3}\sqrt{2gh}\right) = 2MV_B' + 3MV_C'$ 1

Energy conservation $\quad \tfrac{1}{2}(2M)\tfrac{4}{9}(2gh) = \tfrac{1}{2}(2M)V_B'^2 + \tfrac{1}{3}(3M)V_C'^2$ 1

Simplifying these equations

$$\tfrac{4}{3}\sqrt{2gh} = 2V_B' + 3V_C' \quad (5)$$

$$\tfrac{8}{9}(2gh) = 2V_B'^2 + 3V_C'^2 \quad (6)$$

Subs for B' from (5) into (6)

$$\tfrac{8}{9}(2gh) = \tfrac{2}{4}\left(\tfrac{4}{3}\sqrt{2gh} - 3V_C'\right)^2 + 3V_C'^2$$

Q6

(ii) $\frac{16}{9}(2gh) = \frac{1}{2}\left(\frac{16}{9}(2gh) - 8\sqrt{2gh}\,V_c' + 9V_c'^2\right) + 3V_c'^2$

$= \frac{16}{9}(2gh) - 4V_c'\sqrt{2gh} + \frac{9}{2}V_c'^2 + 3V_c'^2$ 3

Giving

$4\sqrt{2gh}\,V_c' = V_c'^2\left(\frac{9}{2}+3\right) = \frac{15}{2}V_c'^2$

As $V_c' \neq 0$

$\underline{V_c' = \frac{8}{15}\sqrt{2gh}}$ (7)

So C rises to a height H given by, using conservation of energy

$3MgH = \frac{1}{2}\,3MV_c'^2$ 1

From (7) $= \frac{3}{2}M\left(\frac{64}{225}(2gh)\right)$

$\underline{H = \left(\frac{64}{225}\right)h}$ 1

12

Q7

(a) There is no contribution from the rotation of the Earth for polar orbit

Using usual notation, if satellite has mass m and speed v,

$$\frac{mv^2}{R_E} = \frac{GM_E m}{R_E^2}$$

$$v^2 = \frac{GM_E}{R_E}$$ 1

However

$$g = \frac{GM_E}{R_E^2}$$ 1

So

$$v^2 = gR_E$$

$$v = \sqrt{gR_E}$$

$$= \sqrt{(9.81)(6.38)\times 10^6}$$ 1

$$v = 7.91 \text{ kms}^{-1}$$ 1

4

(b) For equitorial orbit, speed of Earth can supplement velocity if launched in direction of rotation of the Earth

Speed of rotation of the Earth, v, given by period of rotation

$$\frac{2\pi R_E}{v} = 24\times 60\times 60$$

$$v = 2\pi R_E / 24\times 60\times 60$$

$$v = 0.46 \text{ kms}^{-1}$$ 2

Equitorial launch speed $(7.91 - 0.46) = 7.45 \text{ kms}^{-1}$ 1

Thus ratio

$$\frac{\text{Polar launch speed}}{\text{Equitorial launch speed}} = \frac{7.91}{7.45}$$

$$= 1.06$$ 1

4

Q1

(c) Escape Speed V

The satellite must have sufficient energy to escape from the gravitational field of the Earth

$$\frac{1}{2}mV^2 = \frac{GM_E m}{R_E}$$ 1

$$V = \frac{2GM_E}{R_E}$$

As $g = \frac{GM_E}{R_E^2}$,

$$= \sqrt{2gR_E}$$ 1

Substituting for g and R_E

$$V = 11.2\ \text{kms}^{-1}$$ 1

Minimum Speed

Using Earth rotation for an equatorial launch this can be reduced by $0.46\ \text{kms}^{-1}$ giving 1

$$V = 10.7\ \text{kms}^{-1}$$

4

(d) In order to hit the Sun the speed of probe must be reduced to zero relative to the Sun, so that it 'falls' into the Sun and does not orbit it.

So one must launch it in the opposite direction to the rotation of the Earth around the Sun with a velocity opposite to that of the Earth around the Sun 1

Speed of Earth around the Sun V given by

$$\frac{2\pi R_{ES}}{V} = 365 \times 24 \times 60 \times 60$$

where R_{ES} is Earth–Sun distance 2

$$V = \frac{2\pi(1.50 \times 10^8)}{365 \times 24 \times 60 \times 60}\ \text{kms}^{-1}$$

$$V = 29.9\ \text{kms}^{-1}$$ 1

4

Q7

(a) If the launch speed is v then the launch speed must be sufficient to escape from the gravitational field of the Sun ie for probe of mass m

$$\frac{1}{2}mv^2 = \frac{GmM_S}{R_{ES}} \quad (M_S \text{ mass of Sun})$$ 1

$$v^2 = \frac{2GM_S}{R_{ES}}$$

$$= \frac{2(6.67\times10^{-11})(1.99\times10^{30})}{1.50\times10^{11}} \ (ms^{-1})$$

$$= 17.7\times10^{8}$$

$$v = 42.1\times10^{3}\ ms^{-1}$$

$$v = 42.1\ kms^{-1}$$ 1

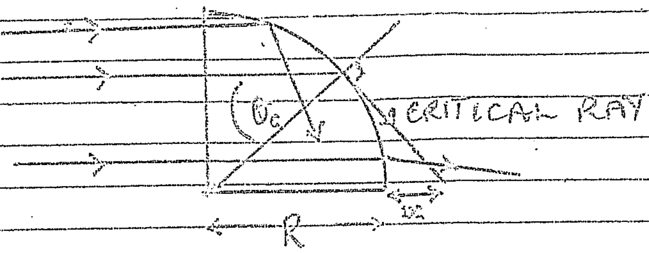
One can use the rotational speed of the Earth to reduce the launch speed by 29.9 kms⁻¹ giving a launch speed in the direction of rotation of the Earth around the Sun of v' given by 1

$$v' = (42.1 - 29.9)\ kms^{-1}$$

$$v' = 12.2\ kms^{-1}$$ 1

4

Q8 (i)



Critical angle θ_c

The angle of incidence, on the curved surface, increases with height of the incident ray. The critical ray has an angle of refraction of $90°$. Subsequent light rays are totally internally reflected, so no light is refracted. Thus for these rays no light reaches the table. 2

The critical ray, for refractive index of prism of μ, is given by

$$\sin\theta_c = \frac{1}{\mu} = \frac{1}{1.5} \quad (1)$$ 1

Using geometry for critical ray in diagram

$$\frac{R}{R+x} = \cos\theta_c \quad (2)$$ 1

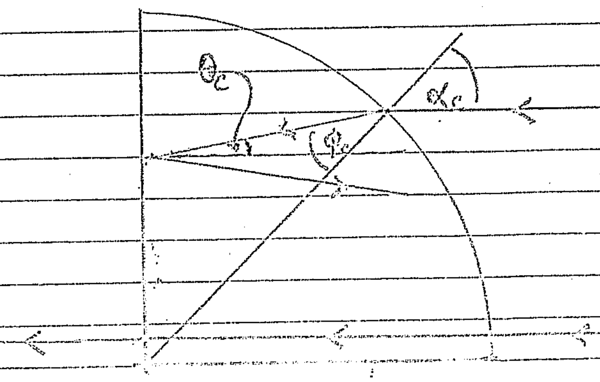
Substituting (1) into (2) with $R = 5.00$ cm

$$x = 1.71 \text{ cm}$$ 1

~~Beyond~~ Below this value of x no light is incident on the table; light reaches the table for ~~$0 \leq x \leq 1.71$~~ $x \geq 1.71$ 1

6

(ii) Let θ_c be the critical angle, angle of incidence on curved face α_c with an angle of refraction ϕ_c (see diagram)



Q8 (ii) For angles of incidence on the curved surface greater than α_c, no light will emerge from the vertical face

Now
$$\mu = \frac{\sin\alpha_c}{\sin\phi_c}$$ 1

and
$$\alpha_c = \theta_c + \phi_c \quad \text{ie} \quad \phi_c = \alpha_c - \theta_c$$

Also
$$\mu = \frac{\sin\alpha_c}{\sin(\alpha_c - \theta_c)} \quad (3)$$ 2

and
$$\sin\theta_c = \frac{1}{\mu}$$

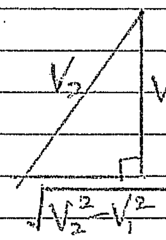
Substituting $\alpha_c = 83{\cdot}27^\circ$ and $\theta_c = 41{\cdot}81^\circ$
plus $\mu = 1{\cdot}5$ into (3)

LHS = 1·50 and RHS = 1·50

Thus for angles of incidence greater than $83{\cdot}27^\circ$ no light emerges from the vertical face of the prism. 1

4

(b) θ is the critical angle



$$\sin\theta = \frac{V_1}{V_2}$$ 1

and
$$\cos\theta = \frac{\sqrt{V_2^2 - V_1^2}}{V_2}$$ 2

Equating times for the two paths

$$\frac{x}{V_1} = \frac{2h}{V_1\cos\theta} + \frac{1}{V_2}\left(x - 2h\tan\theta\right)$$ 3

$$x\left(\frac{1}{V_1} - \frac{1}{V_2}\right) = \frac{2h}{\cos\theta}\left(\frac{1}{V_1} - \frac{\sin\theta}{V_2}\right)$$

$$x\left(\frac{V_2 - V_1}{V_1V_2}\right) = \frac{2hV_2}{\sqrt{V_2^2 - V_1^2}}\left(\frac{1}{V_1} - \frac{V_1}{V_2^2}\right)$$

$$= \frac{2hV_2}{\sqrt{V_2^2 - V_1^2}}\left(\frac{V_2^2 - V_1^2}{V_1V_2^2}\right) = \frac{2h\sqrt{(V_2 - V_1)(V_2 + V_1)}}{V_1V_2}$$ 3

Thus
$$x = 2h\sqrt{\frac{V_2 + V_1}{V_2 - V_1}}$$ 1

6