Q1 2011 BPhO Paper 2

(a) $PV = nRT$

$(10)V = nR(288)$ ① 1

$PV = \frac{1}{2}nR(273+65)$

$P = \frac{1}{2}\frac{nR}{V}(338)$ 1

$= \frac{1}{2}\left(\frac{10}{288}\right)(338)$ from ①

$P = 5.87$ atmospheres 1

3

(b) $R_{AB} = \left(\frac{1}{R_1}+\frac{1}{R_2}\right)^{-1} + R_3$

$\therefore R_1 = \frac{R_1R_2}{R_1+R_2} + R_3$

$R_1(R_1+R_2) = R_1R_2 + R_3(R_1+R_2)$

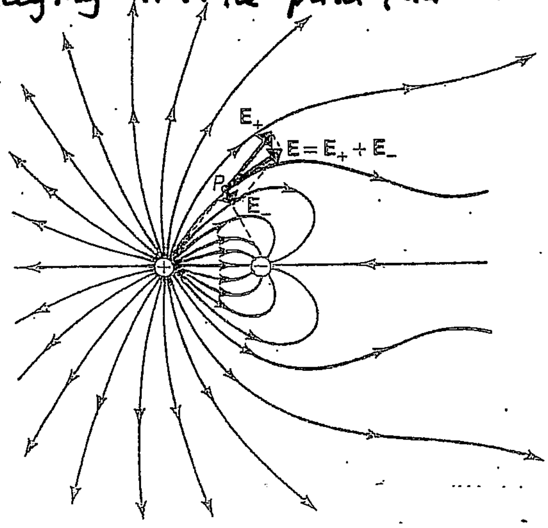
$R_1^2 = R_3(R_1+R_2)$

$R_3 = \frac{R_1^2}{R_1+R_2}$ 1

2

(c) An electric field line is a line that at every point along it the tangent is parallel to the field vector at that point. 2

(give 1 mark for saying it is the path that a charged particle would take)



field lines +ve → -ve 1

shape 1

more from +ve than -ve lines 1

Diagram 3

5

(d) This is the equal to the tension in the cable at the support. If T is the tension and M is the mass of the cable, the equilibrium of the cable requ

(i) $2T\sin 30 = Mg$ 1

$2T(\frac{1}{2}) = 100(9.81)$

$T = 981\text{ N}$ at $30°$ to horizontal (1+1) = 2

Q1

(d) (ii) At the lowest point let tension be T_0. By symmetry it is horizontal. 1
Resolving horizontally for one half of the cable 1

$$T\cos 30 = T_0$$
$$T_0 = 981\left(\frac{\sqrt{3}}{2}\right)$$
$$T_0 = 850\ N$$ 1

5

(e)(i) Rotation speed in radians/min $= \frac{2\pi}{24\times 60}$ 1

Speed of spot $= \frac{2\pi}{24\times 60}\ 10^{3}$ cms/min 1

$=$ ~~43.6~~ 4.36 cms/min 1

(Also 0.0436 m/min or 2.62 m/hr)

(give full marks for dist = 10 tan θ m)

(ii) Image in mirror 20.0 m from hole in ceiling
Speed of spot is twice that in ans. to (i) $=$ 8.73 cms/min 1

4

(f)
(i) Electrons ~~(and X-rays)~~, anti (electron) neutrinos $\frac{1}{2}+\frac{1}{2} = 1$
~~(any two particles = 1 mark)~~

(ii) 20 protons $\frac{1}{2}$
22 neutrons $\frac{1}{2}$
19 electrons 1

3

(g) Condition for constructive interference, as there is an addition phase change of π at air-glass surface,

$$2t\mu = (n+\tfrac{1}{2})\lambda$$ 2

$$2(1.52)(0.42\times 10^{-6}) = (n+\tfrac{1}{2})\lambda$$

$$\lambda = \frac{2(1.52)(0.42\times 10^{-6})}{(n+\frac{1}{2})}$$ 1

$n=1$ gives $\lambda_1 = 851$ nm $\frac{1}{2}$
$n=2$ gives $\lambda_2 = 511$ nm 1
$n=3$ gives $\lambda_3 = 365$ nm $\frac{1}{2}$

The only visible wavelength is $\lambda_2 = 511$ nm 1

6

Q1 3

(b)(i) Wavelength, λ, is twice length of chord $= 2{\cdot}4\,m$ 1

Mass per unit length $m = \frac{0{\cdot}13}{1{\cdot}2} = 0{\cdot}1083\ kg\,m^{-1}$ $\frac{1}{2}$

Tension in chord $T = 50g = 50(9{\cdot}81)\,N$ $\frac{1}{2}$

Frequency, f, given by

$$f\lambda = \sqrt{\frac{F}{m}} = \sqrt{\frac{50(9{\cdot}81)}{0{\cdot}1083}}$$ 1

$$f = \frac{1}{2{\cdot}4}\sqrt{\frac{50(9{\cdot}81)}{0{\cdot}1083}}$$

$$\underline{f = 28\ Hz}$$ 1

Period $\underline{T = \frac{1}{28} = 0{\cdot}036\,s}$ $\frac{1}{2}$

(ii) Amplitude, A, and max. velocity, v, related by

$$v = A\omega = 2\pi A f$$ 1

$$A = \frac{v}{2\pi f}$$

$$= \frac{15}{2\pi(28)}$$ $\frac{1}{2}$

$$\underline{A = 8{\cdot}5\,cm}$$ 1

(iii) Period $T_0 = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1{\cdot}2}{9{\cdot}81}}$ 1

$$\underline{T_0 = 2{\cdot}2\,s}$$ 1

(iv) $\frac{T}{T_0} = 0{\cdot}016$ 1

Assumption justified as $T_0 \gg T$ 1

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(i) Energy conservation $\frac{1}{2}Mu^2 = \frac{1}{2}Mv^2 + \frac{1}{2}mw^2$ (1) 1

Momentum conservation $Mu = Mv + mw$ (2) 1

Proof

From (1) $M(u^2 - v^2) = mw^2$

$M(u-v)(u+v) = mw^2$ (3) 1

From (2) $M(u-v) = mw$ (4) 1

Sub^n (4) into (3)

$$\underline{u + v = w}$$ 1

Q1

(i) VERIFICATION Alternative method in place of "Proof"

If $u+v=w$, then from ②
$$(u-v) = \frac{m}{M}w \quad ⑥$$
Now let us determine if this is consistent with ①
From ①
$$u^2-v^2 = \frac{m}{M}w^2 \quad ⑦$$
Using ⑤ + ⑥ LHS of ⑦ gives
$$(u^2-v^2) = (u+v)(u-v) = \frac{m}{M}w^2$$
However RHS of ⑦ also gives this result, $\frac{m}{M}w^2$.
Thus with assumption $u+v=w$ the equations for energy and momentum conservation are consistent. 3

(ii) $R = \frac{u-v}{u}$

Sub^g ⑥,
$$R = \frac{mw}{Mu} \quad ⑧$$
1

(iii) Sub^g. $v=w-u$ into ⑥
$$2u-w = \frac{m}{M}w$$ 1
$$2u = \left(\frac{M+m}{M}\right)w$$
Sub^g into ⑧
$$R = \frac{2m}{M+m}$$ 1

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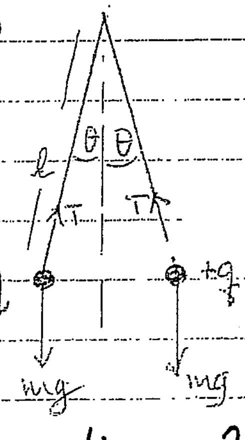
(j) Using notation in the diagram.
Resolving vertically for the forces on one of the balls

$$T\cos\theta = mg = 5.00\times10^{-3}(9.81) \quad ①$$ 1



Resolving horizontally for forces on one ball

$$T\sin\theta = \frac{q^2}{4\pi\varepsilon_0\,(2l\sin\theta)^2}$$
$$= \frac{(1.00\times10^{-6})^2}{4\pi(8.85)\times10^{-12}(2l\sin\theta)^2} \quad ②$$

diag 2
equ ② 1

P1

(j) Dividing ② by ① and substituting $l = 1.00\,m$

$$\cancel{\tan\theta}\sin^2\theta = \frac{(1.00\times10^{-6})^2}{4\pi(8.85\times10^{-12})\,4\,(5.00\times10^{-3})\,(9.81)}$$ 1

$$= 0.0459$$

Substituting $\theta = \frac{1}{2}(41.0^\circ) = 20.5^\circ$ LHS = RHS 1

Thus $\theta = 20.5^\circ$ is a solution 1

7

Q1 (k) Pressure of nitrogen gas $= (75.9 - 62.2) = 13.7$ cms of Hg 1

Volume of gas $= 0.080\,A$ A area of cross sect

$$= 0.080\,\pi\left(\frac{0.0065}{2}\right)^2$$ 1

If there are n mols of nitrogen gas, gas equation gives

$$\left(\frac{13.7}{100}\right)\rho g\,(8+13.7)10^{-2}\,\pi\left(\frac{0.0065}{2}\right)^2 = nR(273+15)$$ 2

ρ density of Hg

$$(1.37\times10^{-1})(1.35\times10^{4})(9.81)\,(21.7)10^{-2}\,\pi\left(\frac{0.0065}{2}\right)^2 = nR(273+1\ldots$$

$$= nR(288)$$

$$(1.37\times10^{-1})(1.35\times10^{4})(9.81)(21.7\times10^{-2}\pi)\left(\frac{0.0065}{2}\right)^2 = 288(8.31)\,n$$

This gives

$$n = 5.46\times10^{-5}$$ 1

Mass of nitrogen $= 14n$ g 1

$$= 7.6\times10^{-4}\,g$$ 1

7

(l)(i) ENERGY $= \frac{1}{2}C(2E)^2 = 2CE^2$ 1

(ii) Energy used = charge × potential

$$= (2CE)\times(2E) = 4CE^2$$ 1

Energy lost $= 4CE^2 - 2CE^2$

$$= 2CE^2$$ 1

(iii) CHARGING IN TWO STAGES

TOTAL WORK DONE IN 1ST STAGE $= \Delta Q(E)$

$$= (CE)(E) = CE^2$$ 1

Q1

(l)(iii) For second stage

Work done $= (\Delta Q)(2E)$
$= (CE)(2E)$
$= 2CE^2$ 1

Total work done $= CE^2 + 2CE^2 = 3CE^2$ 1

Thus energy lost $= 3CE^2 - 2CE^2$
$= \underline{CE^2}$ 1

Thus less energy lost by charging in two stages 1

8

(m) Frequencies given by

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m_1}} \qquad f_2 = \frac{1}{2\pi}\sqrt{\frac{k}{m_2}}$$ 1

$$= \frac{1}{2\pi}\sqrt{\frac{39.48}{0.90}} \qquad = \frac{1}{2\pi}\sqrt{\frac{39.48}{1.10}}$$

$$= 1.054 \text{ Hz} \qquad = 0.953 \text{ Hz}$$ 2

Beat frequency $= 1.054 - 0.953 = 0.10$ Hz $\frac{1}{2}$

Beat period $T = \frac{1}{0.10} = 10$ s $\frac{1}{2}$

4

(n) Let n be the number of complete revolutions, then equating heat generated to work done:

$$(0.4)(0.35\times10^3)5 = 20(0.25)n$$ 2

$$n = 140$$ 1

3

(o) Atmospheric pressure = weight of atmosphere above a square metre

$1.01\times10^5 = 1.23\,tg$ t = height of atm

$$t = \frac{1.01\times10^5}{1.23(9.81)}$$

$$t = 8.37\times10^3 \text{ m}$$

2

Q7

give full marks for putting ϕ as an energy ie not a voltage
ie. putting ϕ instead of $e\phi$.

(a) (i) The photoelectric effect occurs when a photon of frequency f, and energy hf, interacts with an electron in a metal, often an alkali metal, giving up its energy to the (1) electron. If the electron has sufficient energy to overcome the potential barrier of the metal, which for a metal with work function φ is $e\varphi$, it will 'escape' from the metal. Thus photons, of light or EMR, incident on a metal, with appropriate frequencies, can expel electrons from the metal. The incident photon requires an energy

$$hf > e\varphi \qquad hf > W \text{ work function (2)}$$

to expel an electron from the metal, with v, the electron's velocity, being greater than zero. If however

$$e\varphi > hf$$

no electrons are emitted. The critical frequency is f_0, where $hf_0 = e\varphi$. 3

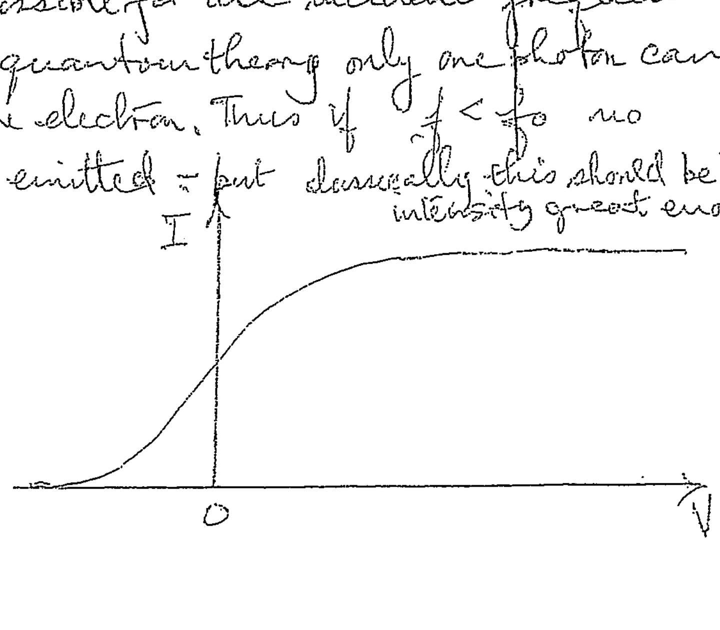
(ii) Conservation of energy requires, for an electron,

$$\frac{1}{2} m_e v^2 = hf - e\varphi = eV_s$$

stopping potential 2

(iii) Classically EMR is a wave, not a beam of photon particles. The electron can, according to the wave theory, absorb any quantity of EMR. When it has absorbed sufficient to overcome the potential barrier $e\varphi$, it will escape from the metal. This is possible for all incident frequencies. 2
Note that in the quantum theory only one photon can interact with one electron. Thus if $f < f_0$ no electrons can be emitted – but classically this should be possible if amplitude/intensity great enough

(iv)



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Q7

(a) (v) From (ii) $\frac{1}{2}m_e v^2 = hf - e\varphi$

As $f = \frac{c}{\lambda}$, this becomes $\frac{1}{2}m_e v^2 = \frac{hc}{\lambda} - e\varphi$ 1

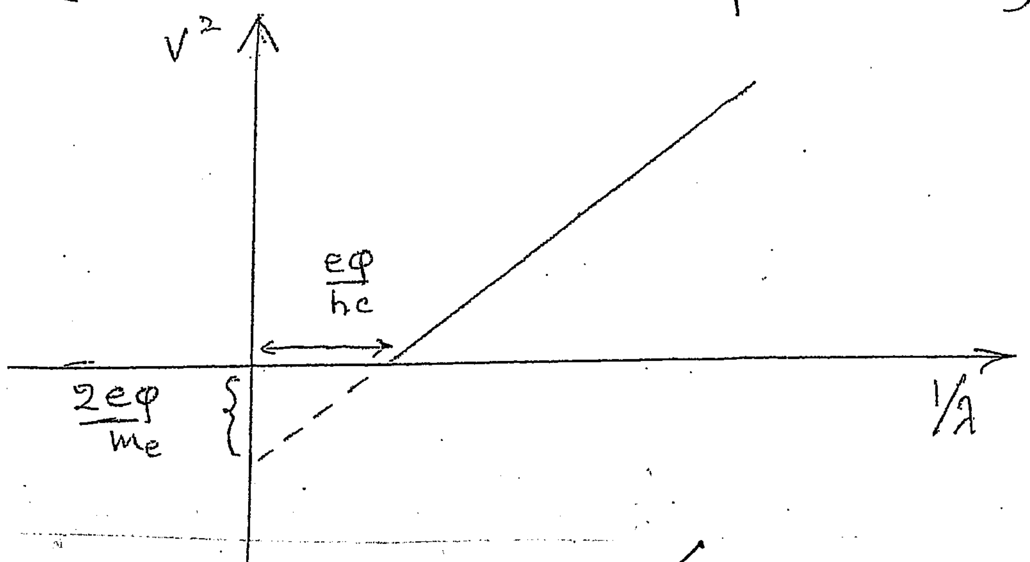
Plotting v^2 against $\left(\frac{1}{\lambda}\right)$ gives a straight line 1

gradient $\left(\frac{2hc}{m_e}\right)$

and intercept $\left(-\frac{2e\varphi}{m_e}\right)$

on the v^2-axis, or $\left(\frac{e\varphi}{hc}\right)$

on the $\left(\frac{1}{\lambda}\right)$ axis. Hence determine φ 1



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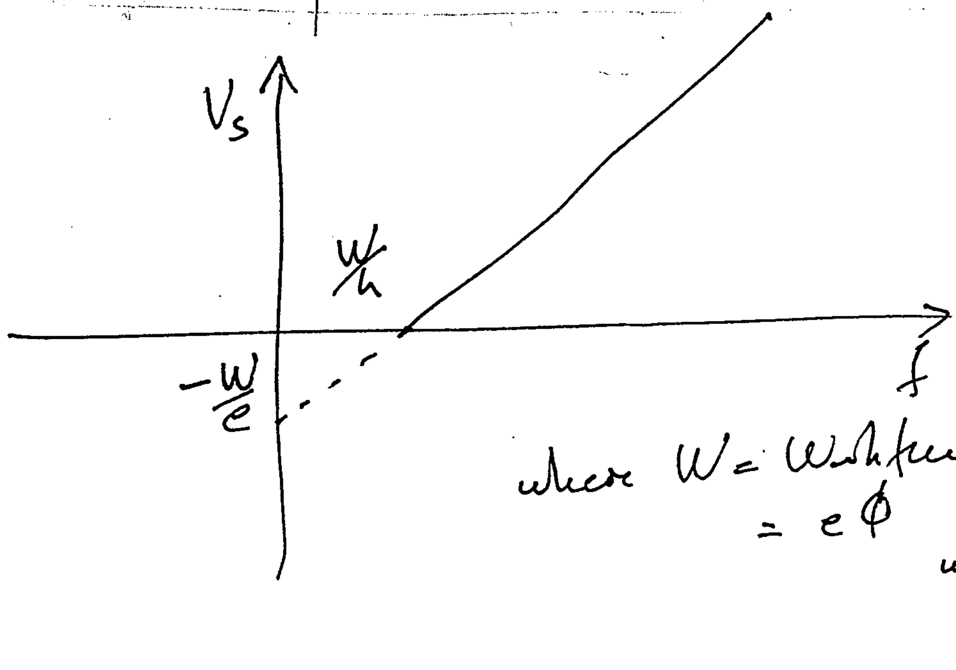
or.



where W = Work function (energy) $= e\phi$ where ϕ is a potential.

Q7

(b) Plot a graph of V_s against f.

$$eV_s = hf - e\varphi \qquad (1)$$

MARKS

Graph correctly drawn, producing a straight line 2

In terms of threshold frequency f_0, (1) gives

$$eV_s = hf - hf_0.$$
$$V_s = \left(\frac{h}{e}\right)f - \left(\frac{h}{e}\right)f_0$$ 1

∴ Gradient $\left(\frac{h}{e}\right)$ and $f = f_0$ when $V_s = 0$ (Also $V_s = \frac{h}{e}f_0$ when f

$f_0 = (4\cdot4 \pm 0\cdot1)10^{14}$ Hz — Correct value 1

Error estimate 1

GRADIENT $\frac{\Delta V_s}{\Delta f} = (4\cdot0 \pm 0\cdot1)10^{15}$ ~~Vs~~ V.s — Correct value 1

Error estimate 1

$h = e\frac{\Delta s}{\Delta f} = 6\cdot4 \times 10^{-34}$ Js — Correct value 1

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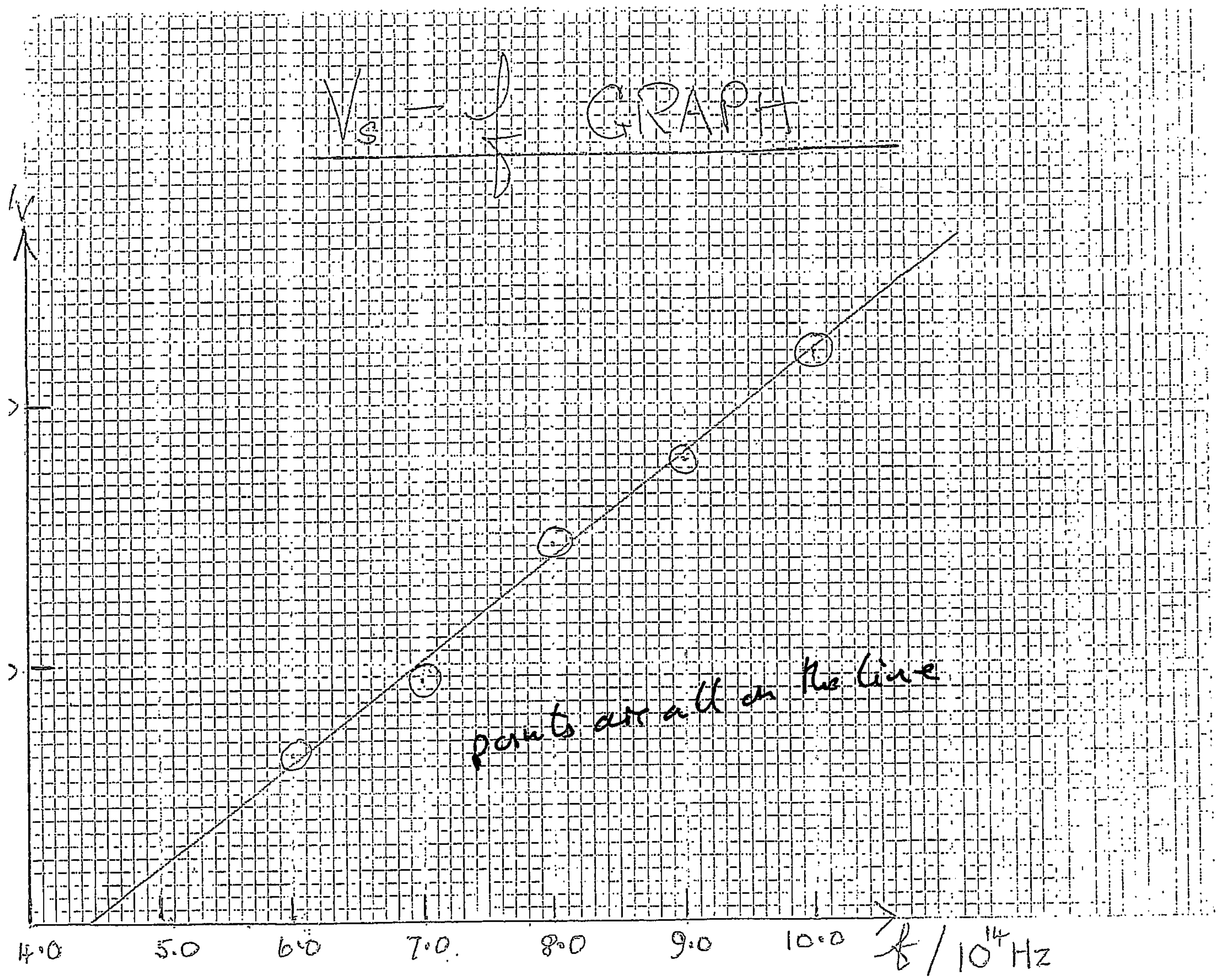
Q7



$V_s - f$ GRAPH
V
4·0
5·0
6·0
7·0
8·0
9·0
10·0
$f / 10^{14}$ Hz
points are all on the line

Q8

(a) If v_0 speed in orbit then for circular motion

$$\frac{mv_0^2}{(R_E+l)} = \frac{GM_Em}{(R_E+l)^2} \qquad 1$$

$$v_0^2 = \frac{GM_E}{(R_E+l)} \qquad (1) \qquad \tfrac{1}{2}$$

K.E. in orbit T given by

$$T = \tfrac{1}{2} m v_0^2$$

Total energy in orbit

$$E_0 = \tfrac{1}{2} m v_0^2 - \frac{GM_Em}{(R_E+l)} \qquad 1$$

$$= -\frac{1}{2}\frac{GM_Em}{(R_E+l)} \qquad \text{from } (1) \qquad \tfrac{1}{2}$$

ON IMPACT ENERGY E_I GIVEN BY

$$E_I = \tfrac{1}{2} m V^2 - \frac{GM_Em}{R_E} \qquad 1$$

$$E_I = \tfrac{1}{2} m (2\times10^3)^2 - \frac{GM_Em}{R_E}$$

Energy absorbed E_A

$$E_A = E_0 - E_I$$

$$= -\frac{1}{2}\frac{GM_Em}{(R_E+l)} - \left(\tfrac{1}{2} m (2\times10^3)^2 - \frac{GM_Em}{R_E}\right) \qquad 1$$

$$= -\frac{GM_Em}{2}\left[\frac{1}{R_E+l} - \frac{1}{R_E}\right] - \tfrac{1}{2} m (2\times10^3)^2$$

$$= \frac{(6.67\times10^{-11})(5.97\times10^{24})(500)}{2}\left[\frac{-10^{-7}}{0.743} + \frac{2\times10^{-7}}{0.683}\right] - \frac{500}{2}(2\times10^3)^2$$

$$E_A = 1.47\times10^{10}\ \text{J} \qquad 1$$

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Q8 (b) The minimum initial velocity is that sufficient for the rocket to reach the point Z where the Earth's attraction equals that of the Moon's attraction so that subsequently it is pulled by the Moon's gravitational force, which will be greater than that of the Earth, towards the Moon – leading to a crash landing. Hence at Z ~~the speed~~ is zero. At Z 1

(i)

$$\frac{GM_EM_R}{d_E^2} = \frac{GM_MM_R}{(R_{EM}-d_E)^2}$$ 2

$$\frac{R_{EM}-d_E}{d_E} = \left(\frac{M_M}{M_E}\right)^{\frac{1}{2}}$$ 1

$$= 0.111$$

$$1.111\, d_E = R_{EM} = 3.83\times10^{8}$$ 1

$$\underline{d_E = 3.45\times10^{8}\,m}$$

Q8 (b) (ii) Conservation of energy initially and at Z gives,

$$\tfrac{1}{2}M_Rv^2 - GM_RM_E\left(\frac{1}{R_E}\right) - GM_RM_M\left(\frac{1}{R_{EM}-R_E}\right)$$ 2

$$= -GM_R\left[\frac{M_E}{d_E} + \left(\frac{M_M}{R_{EM}-d_E}\right)\right]$$ 2

$$v^2 = 2GM_E\left[\frac{1}{R_E}-\frac{1}{d_E}\right] - 2GM_M\left[\frac{1}{R_{EM}-d_E} - \frac{1}{R_{EM}-R_E}\right]$$

$$= 2(6.67\times10^{-11})(5.97\times10^{24})\left[\frac{1}{6.38\times10^{6}} - \frac{1^{*}}{3.45\times10^{8}}\right]$$

$$- 2(6.67\times10^{-11})(7.35\times10^{22})\left[\frac{1^{*}}{3.84\times10^{8}-3.45\times10^{8}} - \frac{1^{*}}{3.84\times10^{8}-6.38\times10^{6}}\right]$$

$$= 2(6.67\times10^{-11})\{(5.97\times10^{24})(1.56\times10^{-7} - 2.90\times10^{-9}) - (7.35\times10^{22})(2.56\times10^{-8} - 2.65\times10^{-9})\}$$

$$= 1.334\times10^{-10}\{(5.97\times10^{24})(1.53\times10^{-7}) - (7.35\times10^{22})(2.29\times10^{-8})\}$$

$$= 1.334\times10^{-10}\{9.13\times10^{17} - 1.68\times10^{15}\}$$

$$= 1.334\times10^{-10}(9.11\times10^{17}) = 1.22\times10^{8}$$ 3

$$\underline{v = 1.1\times10^{4}\ ms^{-1}}$$ 1

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* These terms may be neglected, after justification

Q9

(a) Condition for radioactive equilibrium

$$N_{uranium}\lambda_{uranium} = N_{radium}\lambda_{radium}$$ 1

$$\lambda_U = \frac{\ln 2}{1.4\times10^{17}} \qquad \lambda_{Ra} = \frac{\ln 2}{5.1\times10^{10}}$$

$$N_U = 1.0\times10^{23}$$ 1

$$N_{Ra} = N_U \frac{5.1\times10^{10}}{1.4\times10^{17}}$$

$$= 10^{23}\ \frac{5.1}{1.4}\ 10^{-7}$$

$$N_{Ra} = 3.6\times10^{16}$$ 1

3

Q9 (b)

(i) Number emitted through a solid angle of subtended by the area of $A = 4.0\times10^{-4}\,m^2$ is 60/min

Consequently total emitted $N = 60\times\frac{\text{area of surface of sphere radius } 2.0\,m}{4\times10^{-4}}$

$$N = \frac{4\pi(2.0)^2}{4.0\times10^{-4}}(60) = 240\pi(10^4)\ /\text{min}$$ 1

$$N = 7.5\times10^{6} \text{ per min}$$ 1

(ii) No emitted through area A at 1.8 m $N_{1.8} = \frac{kA}{(1.8)^2}$ where k is a const

But number detected is same as at 2.0 m, $N_{2.0} = \frac{kA}{(2.0)^2}$ 1

Thus % number absorbed by sheet

$$100\times\frac{\frac{kA}{(1.8)^2}-\frac{kA}{(2.0)^2}}{\frac{kA}{(1.8)^2}} = 100\left(1-\left(\frac{1.8}{2.0}\right)^2\right)$$ 1

$$= 100(1-0.81)$$

$$= 19\%$$ 1

9 (b) (ii) Alternatively:

If 60 photons observed at 1·8 m with absorber present, total number observed if "spherical" absorber present, taking into account all directions is

$$N' = \frac{4\pi (1\cdot8)^2 \; 60}{4 \times 10^{-4}} = 6\cdot11 \times 10^6$$ 2

Thus % of γ rays absorbed

$$100 \times \frac{N-N'}{N} = 100 \times \frac{7\cdot5 - 6\cdot1}{7\cdot5} = 19\%$$ 1

5

(c) (i)

$N = N_0 e^{-\lambda t}$

$\dot{N} = \frac{dN}{dt} = -N_0 \lambda e^{-\lambda t}$ (1) 1

$\lambda = \frac{\ln 2}{5600}$ years^{-1} 1

$\lambda = 1\cdot24 \times 10^{-4}$ years^{-1} 1

$N_0 = 6\cdot02 \times 10^{23} \left(\frac{4}{12}\right) \frac{1\cdot25}{10^{12}}$ 1

$= 2\cdot51 \times 10^{11}$

$\therefore \lambda N_0 = 1\cdot24 \times 10^{-4} (2\cdot51 \times 10^{11})$ years^{-1}

$= \frac{1\cdot24 (2\cdot51) 10^{7}}{365 \times 24 \times 60}$ min^{-1} 1

$\lambda N_0 = 5\cdot92 \times 10$ 1

9 (c) Substituting $\dot{N} = 20$ into ①, where T_a is the age in years,

$$20 = 59.1 \exp(-1.24 \times 10^{-4} T_a) \quad ②$$ 1

$$= \exp(-1.24 \times 10^{-4} T_a)$$

$$\ln\left(\frac{20}{59.1}\right) = -1.24 \times 10^{-4} T_a$$

$$\ln(0.338) = -1.24 \times 10^{-4} T_a$$ 1

$$\underline{T_a = 8.74 \times 10^3 \text{ years}}$$ 1

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(ii) EITHER USE CALCULUS OR SUBSTITUTE INTO ORIGINAL EQN.

CALCULUS SOLN.

From ①
$$\Delta\dot{N} = \mp(N_0\lambda)\lambda e^{-\lambda t}\,\Delta t$$

$$= \lambda\dot{N}\,\Delta t$$ 1

$$\Delta t = \pm\frac{\Delta\dot{N}}{\lambda\dot{N}}$$ 1

$$= \frac{(0.4)}{1.24 \times 10^{-4}(20)} \text{ years}$$

$$\underline{\Delta t = \pm 1.6 \times 10^2 \text{ years}}$$ 1

ALTERNATIVELY

SUB. IN ORIG. EQUN $+0.4$ and ΔT_a gives

$$20.0 + 0.4 = 59.1 \exp(-1.24(T_a + \Delta T_a)10^{-4})$$ 1

$$20.4 = 59.1 \exp(-1.24(8.74 \times 10^3 + \Delta T_a)10^{-4})$$

Taking ln
$$T_a + \Delta T_a = \frac{\ln(3.45 \times 10^{-1})}{1.24 \times 10^{-4}} = 8.58 \times 10^3$$ 1

$$\underline{\Delta T_a = 1.6 \times 10^2 \text{ years}} \quad \text{as } T_a = 8.74$$ 1

3