BRITISH PHYSICS OLYMPIAD

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Please note that this mark scheme is a final draft version. It contains several small errors that were corrected during the marking process.

SOLUTIONS BPhO 2010 PAPER 2

Q1

MARK

(a) (i) Energy lost in 8·4 s $= 5 \times 140 \times 3$ J — 1

Rate of loss of energy from lead $= 5\times140\times3 / 8{\cdot}4$ Js^{-1}

$= 250\,W$ — 1

(ii) If specific latent heat of fusion is L then

$3L = 250 \times 300$

$L = 2{\cdot}5 \times 10^4\ J\,kg^{-1}$ — 1

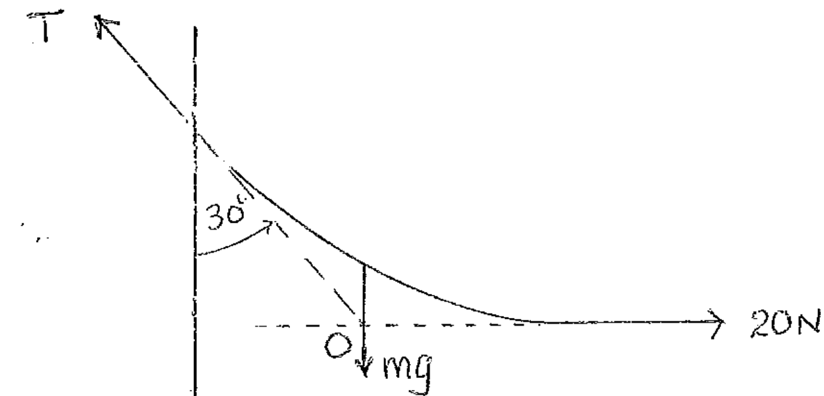
(iii) If specific heat capacity of liquid lead is s then

$3(5)s = 250 \times 10$

$s = 167\ J\,kg^{-1}K^{-1}$ — 1

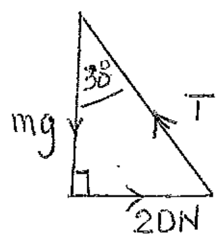
[4]

(b)



Vertical [illegible] of rope must pass through O

Triangle of forces



1

$$\tan 30^\circ = \frac{20}{mg}$$

$mg = 20 \cot 30^\circ$ — 1

$= 20\sqrt{3}$

$= 34{\cdot}64$ N

$m = 3{\cdot}53$ kg — 1

[3]

(c) Surface area of a bit $= \dfrac{\pi\,(5.5^2 - 2.2^2)}{650 \times 10^6 \times 8} = \dfrac{79.8}{5.2 \times 10^9}\ \text{cm}^2$ 2

$= 1.5 \times 10^{-8}\ \text{cm}^2 = 1.5 \times 10^{-12}\ \text{m}^2$ 1

3

(d) Area of lens $= \pi\,(10^{-2})^2 = \pi 10^{-4}\ \text{m}^2$

Energy per s (1% of 100W) $= \frac{1}{100}\,100 = 1\ \text{W}$

Time $= 1.5 \times 10^{-2}\ \text{s}$

Energy reaching film $= \dfrac{\pi(10^{-4})}{4\pi\,(10^2)^2}\,(1)\,1.5\times10^{-2}$ 2

$= 3.8 \times 10^{-11}\ \text{J}$ 1

For n photons of frequency ν

$n h\nu = 3.8 \times 10^{-11}$

As "$\nu = \frac{c}{\lambda}$"

$n(6.6\times10^{-34})\ \dfrac{3.00\times10^{8}}{6\times10^{-7}} = 3.8\times10^{-11}$ 1

$n = \dfrac{3.8\times10^{-11}}{3.3\times10^{-19}}$

$n = 1.2\times10^{8}$ (Accept 1.1×10^{8}) 1

[5]

(e) At temperature T K

$\left(\frac{250}{230}\right)^2 20\,[1 + (T-273)\,5.0\times10^{-3}] = 250$ 2

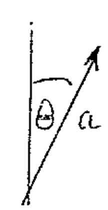
$\left(\frac{230}{250}\right)^2 \frac{1}{20} = 1 + (T-273)\,5.0\times10^{-3}$

$T = 273 + \dfrac{(230)^2}{25}\ \dfrac{1}{5\times10^{-3}}$ 1

$T \approx 2.2\times10^{3}\ \text{K}$ 1

[4]

(f) For amplitude a incident at angle θ



Transmitted amplitude $= a\cos\theta$ 1

This has a transmitted intensity $= a^2\cos^2\theta$

Average over all θ, $= \langle a^2\cos^2\theta\rangle = \frac{1}{2}a^2$ 1

Adding over all amps. for all frequencies in white light

intensity $= \frac{1}{2}\sum_i a_i^2 = \frac{1}{2}I$ as $I = \sum_i a_i^2$ 1

[3]

(g) (i) To escape from planet with initial velocity v requires the KE to be at least equal to initial gravitational P.E.

$$\frac{1}{2}mv^2 = \frac{GmM}{R} \qquad 2$$

$$v = \sqrt{\frac{2GM}{R}} \qquad 1$$

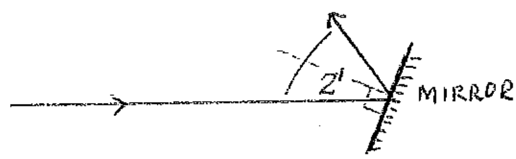
(ii) For planets to have no atmosphere $\langle |v| \rangle$ must be much greater than $\sqrt{\frac{GM}{R}}$ so that they escape from gravitational field

ie. $\langle |v| \rangle \gg \sqrt{\frac{GM}{R}}$ 1

A factor greater than 5 is sufficient

[4]

(h)



1

Mirror rotates through 1' of arc 1

Time taken = $1/(35 \times 360 \times 60)$

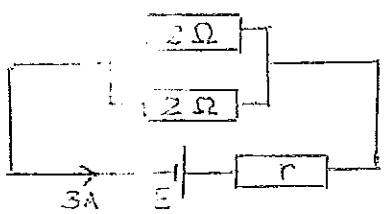
Distance travelled = 400 m

Velocity of light $c = 400 \times 35 \times 360 \times 60 \ \mathrm{ms^{-1}}$

$c = 3.0 \times 10^8 \ \mathrm{ms^{-1}}$ 1

[3]

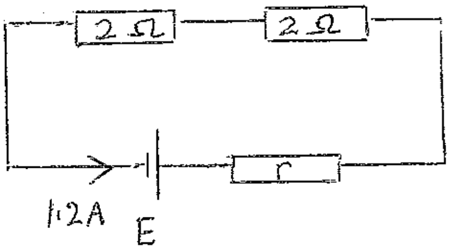
(i) (i)



$$E = 3\left(r + \left(\tfrac{1}{2}+\tfrac{1}{2}\right)^{-1}\right)$$

$$= 3(r+1)$$

$$r = \frac{E}{3} - 1 \qquad 1$$



$E = 1{\cdot}2\,(r+4)$ $\frac{1}{2}$

Sub^g from earlier result

$E = 1{\cdot}2\left(\frac{E}{3} - 1 + 4\right)$ $\frac{1}{2}$

$= 0{\cdot}4E + 3{\cdot}6$

$\underline{E = 6\,V}$ $\frac{1}{2}$

(ii) $r = \frac{E}{3} - 1 = \frac{6}{3} - 1 = 1\,\Omega$ $\frac{1}{2}$

(iii) $P_p = \left(\frac{3}{2}\right)^2 2 = 4{\cdot}5\,W$ $\frac{1}{2}$

$P_s = (1{\cdot}2)^2\,2 = 2{\cdot}88\,W$ $\frac{1}{2}$

[4]

(j) (i) Initial thrust of rocket $= 16 \times 2{\cdot}5 \times 10\,N$

$= 4 \times 10^4\,N$ 1

This is less than weight of rocket $= 9{\cdot}81 \times 4{\cdot}1 \times 10^3 = 4 \cdots \times 10^4\,N$ $\frac{1}{2}$

Consequently rocket does not lift off immediately

(ii) Thrust of rocket is constant $= 4 \times 10^4\,N$

Weight of rocket reduces with time

After time t mass of rocket $= 4{\cdot}1 \times 10^3 - 16t$ $\frac{1}{2}$

Weight of rocket $= 9{\cdot}81(4{\cdot}1 \times 10^3 - 16t)$ $\frac{1}{2}$

Lift off occurs when $4 \times 10^4 = (4{\cdot}1 \times 10^3 - 16t)\,9{\cdot}81$ 1

$\therefore\ 16t = 4{\cdot}1 \times 10^3 - \frac{4 \times 10^4}{9{\cdot}81}$

$= 22{\cdot}5$

$\underline{t = 1{\cdot}4\,s}$ $\frac{1}{2}$

[4]

(k) (i) $\frac{1}{2}$ mark for any two suggestion, for example:

No interactions between molecules
Ideal gas equations
Zero volume of all molecules
Random motion of molecules.
Mean KE $\propto$ absolute temperature ($\langle KE \rangle = \frac{3}{2}kT$) 1

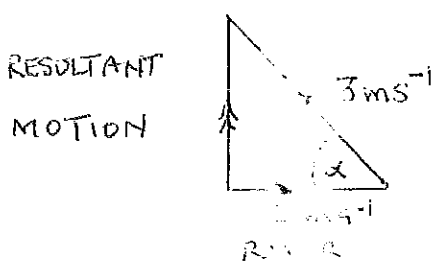
(ii) $\frac{1}{2}$ mark for any two suggestions for example:

Real gas molecules interact
Gas equations deviate from ideal gas laws
Motion no longer purely random
Molecules have finite volume
$\langle KE \rangle \neq \frac{3}{2}kT$ 1

(iii) $\frac{1}{2}$ mark for any two suggestions such as:
Ideal gas satisfies $PV = RT$
Real gas deviates from this relation
(Special cases: Boyles & Charles laws)
Thermodynamic properties deviate from ideal values 1

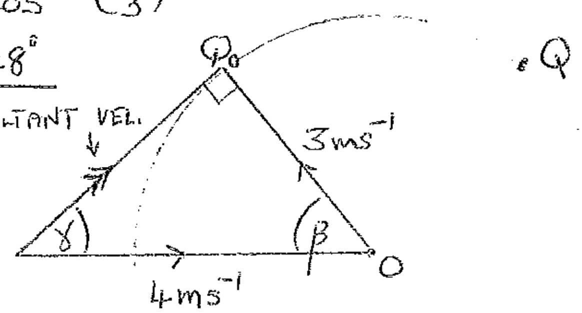
[3]

(l) (i)



To travel perpendicular to bank he has to row at angle α where
$\alpha = \cos^{-1}\left(\frac{2}{3}\right)$ 1
$\underline{\alpha = 48^\circ}$ 1

(ii) CIRCLE CENTRE O



(l) No solution possible perpendicular to bank ie same as velocity of river 1

Possible directions of resultant motion along PQ, where Q is on a circle, centre O, radius $QO = 3\,ms^{-1}$ in vector diagram } 2

The shortest path occurs for largest value of angle γ which will occur when PQ tangential to circle at PQ_0 1

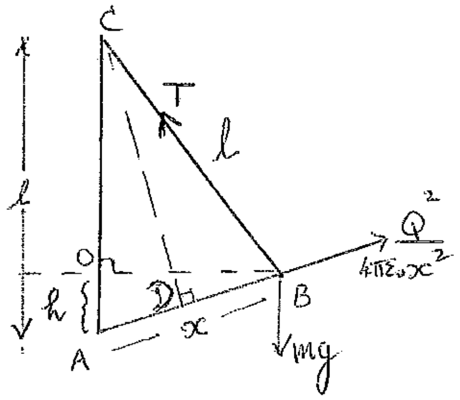
(Lower half circle excluded as it is on land)

$\therefore \sin\gamma = \frac{3}{4}$

$\underline{\gamma = \sin^{-1}\frac{3}{4} = 48.6^\circ}$ 1

[7]

(m)



Isoceles triangle of forces ABC sides l, l and x.
Forces T, mg and $\frac{Q^2}{4\pi\varepsilon_0 x^2}$, where $T = mg$ as ABC isosceles 1

(i) $T = mg$ 1

(ii) $W = mgh + \frac{Q^2}{4\pi\varepsilon_0 x}$ ① 2

From Δ of forces $\frac{x}{l} = \frac{Q^2}{4\pi\varepsilon_0 x^2 mg}$ ② 1

In similar triangles OAB and CAD, where D mid point AB.

$$\frac{h}{x} = \frac{\frac{x}{2}}{l}$$

$x^2 = 2hl$ ③ 1

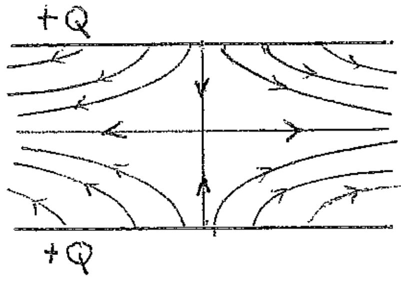
Sub^g ② into ① $\frac{x}{l} = \frac{Q^2}{(4\pi\varepsilon_0) 2hlmg}$ ④ 1

Sub^g for x from ④ into ①

$W = mgh + 2mgh = \underline{3mgh}$ 1

[8]

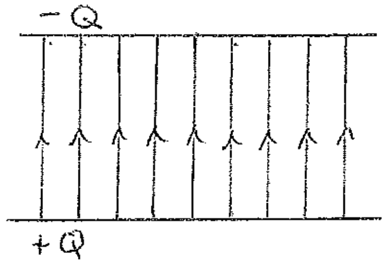
(a) (i)



Drawing 1

Correct arrows 1

(ii)



Drawing $\frac{1}{2}$

Correct arrows $\frac{1}{2}$

(iii) Magnitude of charges same in both cases, (i) and (ii) and distances between all pairs of charges same in both cases. Consequently only difference, in (ii) from (i), is sign of charges. Thus if force of repulsion in (i) is F_+

Force of attracting in (ii) is $F_- = -F_+$

(using inverse square law of force between charges)

Thus $\quad \frac{F_+}{F_-} = -1$ 1

(iv) $C = \frac{\varepsilon_0 A}{x}$ 1

Thus energy $E = \frac{Q^2}{2C} = \frac{Q^2 x}{2\varepsilon_0 A}$ (A) 1

(v) Force of attraction $F = \frac{dE}{dx} = \frac{Q^2}{2\varepsilon_0 A}$

Alternatively comparing with gravitational energy of mass m, mgh, which produces a force mg ie "E = Fx"

Thus from (A) $\quad F = \frac{Q^2}{2\varepsilon_0 A}$ 1

[7]

(o) Let f be natural frequency and natural wavelength λ.
Wavelength λ_s produce by source moving towards observer, velocity V_s, is given by

$$\lambda_s = \lambda - \frac{V_s}{f}$$ 1

Then, for speed of sound c,

$$f_s = \frac{c}{\lambda_s} = \frac{c}{\lambda - \frac{V_s}{f}}$$ 1

$$= \frac{c}{\frac{c}{f} - \frac{V_s}{f}}$$

$$\underline{f_s = f\left(\frac{c}{c - V_s}\right)}$$ 1

Max speed $V_s = 2\pi(1.2)3 = 22.6\ ms^{-1}$ 1
Min speed $-V_s = -22.6\ ms^{-1}$ 1

Thus f is in the range given by

$$f_s = 256\left(\frac{340}{340 \mp 22.6}\right)$$ 1

Thus the variation is $\underline{240 \text{ to } 274\ Hz}$ 1

[7]

(p) (i) Velocity along the field $V_{\parallel} = v\cos\theta$
This remains constant as no force acts in this direction

Velocity perpendicular to field $V_{\perp} = v\sin\theta$ 1
The field B produces a constant force on the electron perpendicular to $V_{\perp}$, which causes it to rotate in a circle, with constant angular velocity, in this plane. 1

(ii) This force is $Bev\sin\theta$ (note constant value)
For motion in a circle radius r

$$\frac{m_e(v\sin\theta)^2}{r} = Bev\sin\theta$$ 1

$$r = \frac{m_e V\sin\theta}{Be}$$ 1

(p) Hence one period T given by

$$T = \frac{2\pi r}{v\sin\theta}$$ 1

$$= \frac{2\pi m_e v\sin\theta}{Bev\sin\theta}$$

$$T = \frac{2\pi m_e}{Be}$$ 1

(iii) Distance travelled in time T, L, is given by

$$L = T(v\cos\theta)$$ 1

$$L = \frac{2\pi m_e v\cos\theta}{Be}$$ 1

[8]

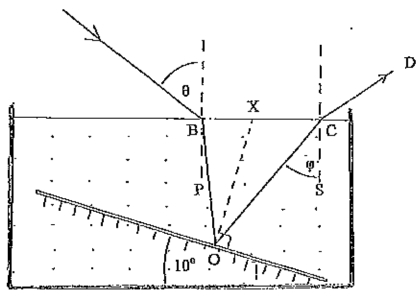
?2 (a) Refractive index $\mu = \dfrac{\text{REAL HEIGHT}}{\text{APPARENT HEIGHT}} = \dfrac{8}{5} = 1.6$ 2

(b) If light emerges at C we require for glass of refractive index n

$n \leq \dfrac{\sin 90^\circ}{\sin\phi} = \dfrac{1}{\sin\phi}$ (not totally internally reflected) 1

$\sin\phi \leq \frac{1}{n} = \frac{3}{4}$

$\underline{\phi \leq 48.59^\circ}$ 1



From diagram, $\angle OCB = 90^\circ - \phi$ $\angle OXC = 90^\circ + 10^\circ = 100^\circ$ as mirror tilted by 10°

In ΔCOX $\angle COX = \phi - 90^\circ$

$\angle BOX = \phi - 90^\circ$ angle of incid. = angle of refl.

In ΔBOX $\angle OBX = 110 - \phi$ angles of Δ add to 180° 2

$\therefore \angle PBO = 90^\circ - (110 - \phi) = \phi - 20$

Refraction at B: $n = \dfrac{4}{3} = \dfrac{\sin\theta}{\sin(\phi - 20)}$

$\sin(\phi - 20) = \frac{3}{4}\sin\theta$

If $\phi < 48.58^\circ$ $\sin(\phi - 20) \leq \sin(28.58^\circ)$

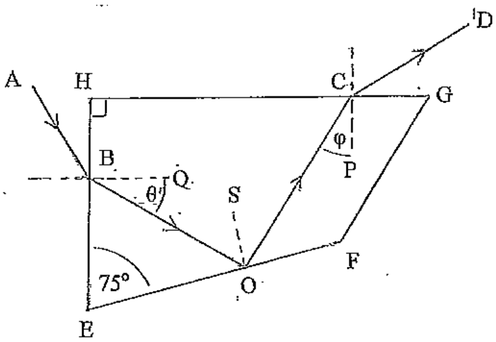
$\therefore \sin\theta \leq \frac{4}{3}\sin(28.59^\circ)$ 2

$\underline{\theta \leq 39.6}$

<u>Maximum θ is 39.6°</u> [6]

P2

(c)



(i)

$$\angle EBO = 90 - \theta$$
$$\angle EOB = 180 - (90-\theta) - 75 = 90 + \theta - 75 = 15 + \theta$$
$$\angle BOS = 90 - \angle EOB = 75 - \theta$$
$$\underline{\angle BOC = 2(75-\theta) = 150 - 2\theta}$$

2

(ii)

$$\angle HCO = 360 - 90 - (90+\theta) - \angle BOC$$
$$= 180 - \theta - (150 - 2\theta)$$
$$= 30 + \theta$$
$$\phi = 90 - \angle HCO = 60 - \theta$$
$$\underline{\phi = 60 - \theta}$$

2

(iii) If $\theta = \phi$

$$2\theta = 60$$
$$\underline{\theta = 30}$$

1

Total internal reflection at O requires

$$n > \frac{\sin 90}{\sin(75-\theta)}$$ ie $\sin 90 / \sin BOS$ 2

$$> 1/\sin 45$$
$$\underline{n > 1.414}$$

1

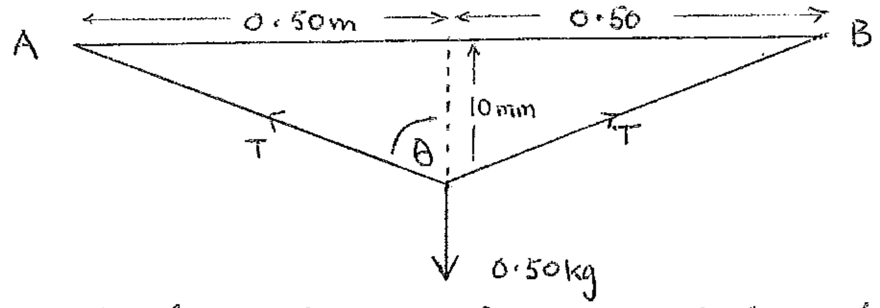
(iv) $\underline{90^\circ}$ as the faces EH and HG are perpendicular and the angles of refraction at these faces are equal, θ and φ equally inclined to the faces, in the same sense 2

[8]

(d) The ripples on the water surface act like a continuous series of mirrors inclined to the horizontal with ± a few degrees. This produces a continuous series of images of the light forming a light 'pillar' ie elongated image of the light source [2]

P3

i)



Increase in length of wire Δl given by, using Pythagoras' theorem,

$$\Delta l = 2\sqrt{(5\times10^{-1})^2 + (10^{-2})^2} - 1.00 \qquad 1$$

$$= 2(5.0\times10^{-1})\sqrt{1+\left(\frac{10^{-2}}{5\times10^{-1}}\right)^2} - 1.00$$

$$= \sqrt{1+\left(\frac{1}{50}\right)^2} - 1.00$$

$$= \frac{1}{2}\left(\frac{1}{50}\right)^2 + \cdots \qquad \text{using Binomial Th or directly} \qquad 1$$

$$= 2.0\times10^{-4}\ \text{m}$$

$$\therefore \frac{\Delta l}{l} = 2.0\times10^{-4} \qquad 1$$

Resolving forces

$$2T\cos\theta = 0.50\,(9.81) \qquad 2$$

$$2T\,\frac{10}{\sqrt{(500)^2+(10)^2}} = 0.50\,(9.81) \qquad 2$$

$$2T = (0.50)(9.81)\sqrt{(50)^2+1}$$

$$T \simeq (0.25)(50)(9.81)$$

$$\simeq 122.6\ \text{N} \qquad 1$$

Thus Young's modulus

$$Y = \frac{\frac{122.6}{10^{-6}}}{2.0\times10^{-4}} \qquad 1$$

$$Y = 6.1\times10^{11}\ \text{Nm}^{-2} \qquad 1$$

[10]

Q3

) (i)

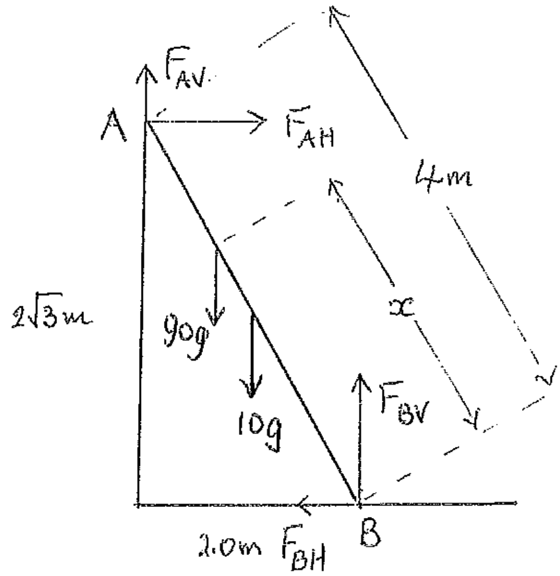


Diagram 2 MARKS ; take $\frac{1}{2}$ off for any missing forces 2

(ii) For smooth wall $F_{AV} = 0$

Resolving vertically $F_{BV} = 90g + 10g = 100g$ 1

Resolving horizontally $F_{BH} = F_{AB}$ 1

Taking moments about A

$$2\sqrt{3}\,F_{OH} + 1(10g) + (4-x)\tfrac{1}{2}(90g) - 2F_{BV} = 0$$ 2

Substituting $F_{BV} = 100g$

$$F_{BH} = \frac{[200 - 10 - (4-x)\frac{1}{2}90]g}{2\sqrt{3}}$$

$$F_{BH} = \frac{(20+90x)\,g}{4\sqrt{3}} \quad (1)$$ 2

If μ coeff. of frict...

Now $F_{BH} \leq \mu F_{BV}$

$\leq 100\mu g$

Thus from (1) $\mu \geq \dfrac{20+90x}{100(4\sqrt{3})}$

For $x = 4.0$m $\mu \geq 0.55$

Minimum value of μ is 0.55 2

[10]

Q4

(i) At equilibrium, with spring constant k,

$$kx_1 = Mg \quad , \qquad \text{where} \quad \omega^2 = \frac{k}{M}$$ 1

Giving

$$x_1 = \frac{Mg}{k} = \frac{Mg}{M\omega^2}$$

$$x_1 = \frac{g}{\omega^2}$$ 1

[2]

(ii)

$$V = \tfrac{1}{2}k(x+x_1)^2 - Mgx$$ 1

$$= \tfrac{1}{2}M\omega^2(x+x_1)^2 - Mgx$$

$$= \tfrac{1}{2}M\omega^2\left(x + \frac{g}{\omega^2}\right)^2 - Mgx$$ 1

$$= \tfrac{1}{2}M\omega^2\left(x^2 + \frac{2gx}{\omega^2} + \left(\frac{g}{\omega}\right)^2\right) - Mgx.$$

$$V = \tfrac{1}{2}M\omega^2x^2 + \tfrac{1}{2}M\left(\frac{g}{\omega}\right)^2 \qquad (A)$$ 1

[3]

V depends on x^2, not x, and $\tfrac{1}{2}M\left(\frac{g}{\omega}\right)^2$ is a constant

(iii) Max. PE occurs when $x = A$, so V_{max} given by

$$V_{max} = \tfrac{1}{2}M\omega^2A_1^2 + \tfrac{1}{2}M\left(\frac{g}{\omega}\right)^2$$

This occurs when velocity stationary. Change in PE from $x = A_1$ 1
to $x = 0$ is from (A)

$$\tfrac{1}{2}M\omega^2A_1^2$$

This is the KE at equilibrium position: $T_1 = \tfrac{1}{2}M\omega^2A_1^2$
(velocity ωA_1)

Thus

$$E_1 = V_{max} = \tfrac{1}{2}M\omega^2A_1^2 + \tfrac{1}{2}M\left(\frac{g}{\omega}\right)^2$$ 1

[2]

(iv) If after collision M moving with velocity V_1 and $\tfrac{1}{2}M$ has velocity V_2
Conservation of momentum

$$M(A_1\omega) = MV_1 + \tfrac{1}{2}MV_2 \qquad \text{ie } A_1\omega = V_1 + \tfrac{1}{2}V_2$$ 1

Conservation of energy

$$\tfrac{1}{2}M(A_1\omega)^2 = \tfrac{1}{2}MV_1^2 + \tfrac{1}{4}MV_2^2 \quad \text{ie } (A_1\omega)^2 = V_1^2 + \tfrac{1}{2}V_2^2$$ 1

From these equations

$$(A_1\omega)^2 = V_1^2 + \tfrac{1}{2}\,4\,(A_1\omega - V_1)^2$$

thus

$$3V_1^2 - 4V_1(A_1\omega) + (A_1\omega)^2 = 0$$

Q4

(iv) Solving,
$$V_1 = \tfrac{1}{6}[4(A_1\omega) \pm \sqrt{16-12}\ (A_1\omega)]$$
$$V_1 = \tfrac{1}{6}(4 \pm 2) A_1\omega$$
$$V_1 = A_1\omega \quad \text{or} \quad \tfrac{1}{3}A_1\omega$$ 2

$V_1 = \frac{1}{3}A_1\omega$ only acceptable solution if changed on collision
Thus amplitude reduces to $A_2 = \frac{1}{3}A_1$, as vel^y = amp. × ω 1

[5]

(v) New equilibrium position, x_2, given by
$$kx_2 = \tfrac{3}{2}Mg$$
$$\underline{x_2 = \frac{3g}{2\omega^2}} \qquad \text{as } k = M\omega^2$$ 1

Conservation of momentum gives new velocity after collision, u,
$$M(A_1\omega) = \tfrac{3}{2}Mu_2$$
$$\underline{u_2 = \tfrac{2}{3}A_1\omega}$$ 1

Potential energy using (A) with M replaced by $(\frac{3}{2}M)$ and $x_2 - x_1 = \frac{g}{2\omega^2}$, measured from new equilibrium position 1
$$P.E. = \tfrac{1}{2}(\tfrac{3}{2}M)\left(\frac{g}{2\omega^2}\right)^2 + \tfrac{1}{2}(\tfrac{3}{2}M)\left(\frac{g}{\omega}\right)^2$$ 1

The KE after collision is $\quad \frac{1}{2}(\frac{3}{2}M)u_2^2 = \frac{1}{2}(\frac{3}{2}M)(\frac{2}{3}A_1\omega)^2$ 1

Equating total energy at zero velocity, amplitude A_2, and following collision, measured from new equilibrium position

$$\tfrac{1}{2}(\tfrac{3}{2}M)\omega^2A_2^2 + \tfrac{1}{2}(\tfrac{3}{2}M)\left(\frac{g}{\omega}\right)^2 = \tfrac{1}{2}(\tfrac{3}{2}M)(\tfrac{2}{3}A_1\omega)^2 + \tfrac{1}{2}(\tfrac{3}{2}M)\omega^2\left(\frac{g}{2\omega^2}\right)^2 + \tfrac{1}{2}(\tfrac{3}{2}M)\left(\frac{g}{\omega}\right)^2$$
$$\tfrac{1}{2}\omega^2A_2^2 = \tfrac{1}{2}\omega^2\left(\frac{g}{2\omega^2}\right)^2 + \tfrac{1}{2}(\tfrac{2}{3}A_1\omega)^2$$ 2

$$\underline{\underline{A_2 = \left[\left(\frac{g}{2\omega^2}\right)^2 + \left(\frac{2A_1}{3}\right)^2\right]^{1/2}}}$$ 1

[8]

Q5

(a) (i) $Q = CV$

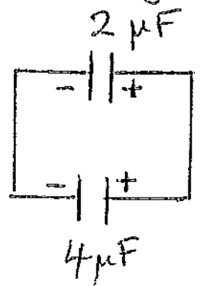
For $2 \cdot 0\,\mu F$ capacitor: $120\times10^{-6} = 2 \cdot 0\times10^{-6} V_1$

$\underline{V_1 = 60V}$ $\frac{1}{2}$

For $4 \cdot 0\,\mu F$ capacitor: $120\times10^{-6} = 4 \cdot 0\times10^{-6} V_2$

$\underline{V_2 = 30V}$ $\frac{1}{2}$

Connecting capacitors



If charge Q on 2μF capacitor and 240−Q on 4μF capacitor (conservation of charge) common potential V_c given by

$$V_c = \frac{Q}{2 \cdot 0\times10^{-6}} = \frac{240-Q}{4 \cdot 0\times10^{-6}}$$ 1

Giving $Q = 80\,\mu C$

and $\underline{V_c = 40V}$ 1

(ii) Energy of a capacitor $= \frac{1}{2}CV^2 = \frac{1}{2}QV$ $\frac{1}{2}$

Initial total energy $= \frac{1}{2}(120\times10^{-6})(60+30) = 5 \cdot 4\times10^{-3}$ J 1

Final energy $= \frac{1}{2}(40)(80+160)10^{-6} = 4 \cdot 8\times10^{3}$ J 1

Energy lost due to heating of 6×10^{-4} J $\frac{1}{2}$

6

Q5

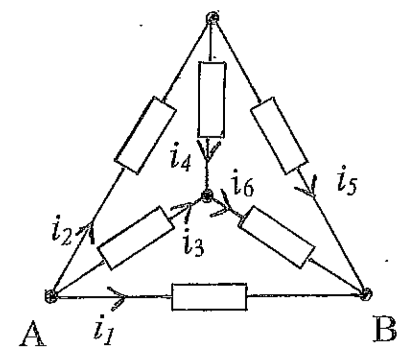
b) (i) The currents are all reversed 1

When V replaced by $-V$ the circuit is just a 'reflection' of the original circuit. So comparing currents in the original circuit and its reflection

$i_2 = i_6$ ½

$i_3 = i_5$ ½

$i_4 = -i_4$ ½



(ii) $i_1 = \frac{V}{R}$ ½

As $i_4 = -i_4$, $i_4 = 0$ ½

So $i_2 = i_6 = \frac{V}{2R}$ 1

and $i_3 = i_5 = \frac{V}{2R}$ 1

(iii) As $i_4 = 0$, $2R$, $2R$ and R are all in parallel with each other

Total resistance across AB $= \left(\frac{1}{2R} + \frac{1}{2R} + \frac{1}{R}\right)^{-1}$ 1

$= \frac{R}{2}$ ½

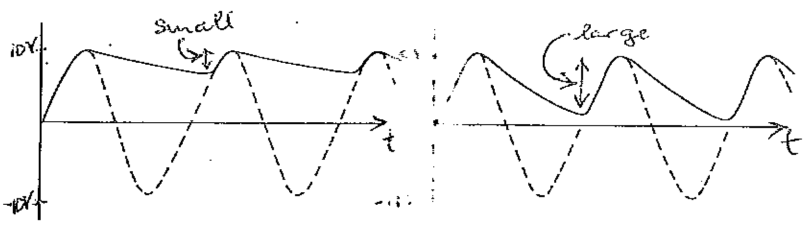
7

(c) (i) Time constant $= CR$

For $10\,k\Omega$ $CR = (50 \times 10^{-6})(10^4) = 0.5\,s$ ½

$CR = (50 \times 10^{-6})(100) = 5.0 \times 10^{-3}\,s$ ½

(ii)

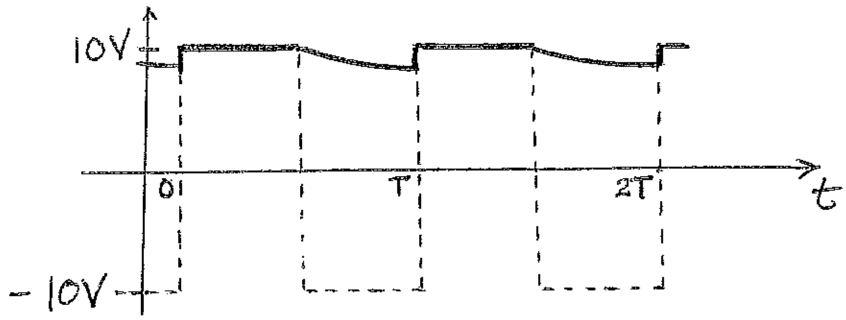


RC = 0.5 s   RC = 0.005 s

1+1 = 2

One mark for each graph correct

5(c) (iii)



Capacitor will discharge for half a period, $\frac{T}{2}$

ie $\frac{1}{2}\left(\frac{1}{50}\right) = 10^{-2}$ s $\frac{1}{2}$

Voltage at time t, $V(t) = 10\,e^{-t/RC}$ 1

At $t = 10^{-2}$ s,

$$V = 10\exp\left(-\frac{10^{-2}}{0.5}\right)$$ $\frac{1}{2}$

$$= 10\exp(-0.02)$$

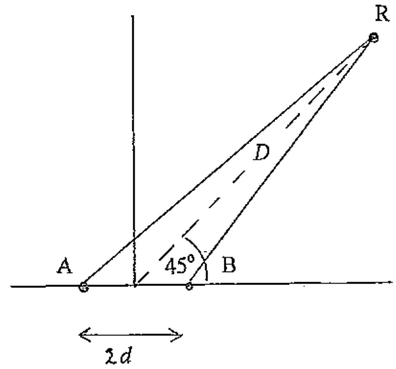
$$= 9.782 \text{ V}$$ 1

Fractional change $= \frac{0.218}{10} \simeq \underline{0.022}$ 1

[7]

Q6

Using cosine rule in diagram,



a)
$$(AR)^2 = D^2 + \left(\tfrac{1}{4}\right)^2 - 2D\left(\tfrac{1}{4}\right)\cos 135^\circ$$
$$= D^2 + \left(\tfrac{1}{4}\right)^2 + \tfrac{1}{2}D\left(\tfrac{1}{\sqrt{2}}\right)$$
$$(AR)^2 = D^2 + \left(\tfrac{1}{4}\right)^2 + \tfrac{1}{4}\sqrt{2}D \qquad 1$$
$$(BR)^2 = D^2 + \left(\tfrac{1}{4}\right)^2 - 2\left(\tfrac{1}{4}\right)D\cos 45^\circ$$
$$(BR)^2 = D^2 + \left(\tfrac{1}{4}\right)^2 - \tfrac{1}{4}\sqrt{2}D \qquad 1$$

(i) $$t_A = \frac{AR}{C_S} = \frac{1}{C_S}\left[D^2 + \left(\tfrac{1}{4}\right)^2 + \tfrac{1}{4}\sqrt{2}D\right]^{\frac{1}{2}} \qquad (1) \qquad 1$$

(ii) $$t_B = \frac{BR}{C_S} = \frac{1}{C_S}\left[D^2 + \left(\tfrac{1}{4}\right)^2 - \tfrac{1}{4}\sqrt{2}D\right]^{\frac{1}{2}} \qquad (2) \qquad 1$$

[4]

b) Frequency emitted at time t is $f = 10t$ 1

(i) Frequency reaching R from A will have been emitted at $(t - t_A)$ 1

Thus " " " " " $f_A = 10(t - t_A)$ (t_A given by (1)) 1

(ii) Similarly for B, $f_B = 10(t - t_B)$ (t_B given by (2)) 1

[4]

c) As $D \gg \frac{1}{4}$ m in (1) and (2), AR and BR are approximately equal to D. Thus f_A and f_B are approximately equal to, say, f_{AB} where 1

$$f_{AB} = 10\left(t - \frac{D}{C_s}\right) \qquad 1$$

The error in 'D' term
$$\Delta D = \pm\, d\cos 45$$
$$= \pm \frac{1}{4}\frac{\sqrt{2}}{2} \text{ m}$$
$$= \pm \frac{1}{8}\sqrt{2} \text{ m} \qquad 1$$

Thus accuracy in f_{AB}
$$\Delta f = \pm 10\left(\frac{\sqrt{2}}{8}\right)\frac{1}{C_S}$$
$$\Delta f = \pm 1.77\left(\frac{1}{C_S}\right) \text{ Hz} \qquad 1$$

4

(d) Wavelength λ given by
$$\lambda = \frac{C_s}{f_{AB}} = \frac{C_s}{10\left(t - \frac{D}{C_s}\right)} \qquad (3) \qquad 1$$

Q6

d) For the first minimum the path difference of the sound reaching R from A and B must differ by $\lambda/2$. 1

So

$$2d\cos 45^\circ = \tfrac{1}{2}\lambda$$ 1

or

$$4d\left(\frac{\sqrt{2}}{2}\right) = \lambda$$

$$= \frac{c_s}{10(t - D/c_s)} \quad \text{from } 3$$ 1

$$4\left(\frac{1}{4}\right)\left(\frac{\sqrt{2}}{2}\right) 10\left(t - \frac{D}{c_s}\right) = c_s$$

As $t = 52.2\,s$ and $D = 1200\sqrt{2}\,m$,

$$10\left(52.2 - \frac{1200\sqrt{2}}{c_s}\right)\frac{1}{\sqrt{2}} = c_s$$ 1

$$396.1 c_s - 1200 = c_s^2$$

$$c_s^2 - 396.1 c_s + 1200 = 0$$ 1

Solving

$$c_s = \tfrac{1}{2}\left[396.1 \pm \sqrt{(396.1)^2 - 4(1200)}\,\right]$$ 1

$$= 363\ ms^{-1} \quad \text{or} \quad 33\ ms^{-1}$$

Only physically acceptable solution is $c_s = 363\ ms^{-1}$ 1

[7]

Q7

(a) If N_0 be the number of atoms initially present and t years the age of the Earth

For U^{238}

$$N_1(t) = N_0 e^{-\alpha t}$$

For U^{235}

$$N_2(t) = N_0 e^{-\beta t}$$

1

Then $$\frac{N_1}{N_2} = 140 = e^{-(\alpha-\beta)t}$$ 1

$$\ln(140) = (\beta-\alpha)t$$
$$= \ln 2\left(\frac{1}{7.13\times10^8} - \frac{1}{4.5\times10^9}\right)t$$ 1

$$\frac{\ln(140)}{\ln(2)} = \frac{1}{10^9}\left(\frac{10}{7.13} - \frac{1}{4.5}\right)t$$ 1

$$7.129 = 10^{-9}(1.4025 - 0.2222)t$$

$$7.129 = 10^{-9}(1.180)t$$ 1

$$\underline{t = 6.04\times10^9 \text{ years}}$$ 1

6

b) $$N = N_0 e^{-\lambda t}$$ where $\lambda = \ln 2/45$ days^{-1} 1

$$N(30) = 4\times10^5 e^{-30\lambda}$$
$$= (4\times10^5)\exp\left(-\tfrac{30}{45}\ln 2\right)$$
$$= (4\times10^5)\exp\left(-\tfrac{2}{3}\ln 2\right)$$
$$= (4\times10^5)\exp\left(-\ln 2^{2/3}\right)$$
$$= (4\times10^5)\, 2^{-2/3}$$

2

$$\underline{N(30) = 2.52\times10^5 \text{ Bq}}$$ 1

FRACTION OF MASS REMOVED

Activity in 5×10^{-3} m^3 of oil

$$\frac{126}{10\times60}\left(\frac{5.0\times10^{-3}}{10^3\times10^{-6}}\right) = \frac{126}{600}(50) = \frac{21}{2} \text{ Bq}$$ 1

Fraction of mass removed $$= \frac{\frac{21}{2}}{2.52\times10^5} = \underline{4.2\times10^{-5}}$$ 1

ie $\underline{4.2\times10^{-3}\ \%}$

6

Q7

c) $$N = N_0 e^{-\lambda t} \quad \text{(A)}$$ $\frac{1}{2}$

where

$$\lambda = \ln 2/70$$
$$= 0.00990 \ s^{-1}$$ $\frac{1}{2}$

Substituting $N_0 = 82$ into RHS of A gives

$$82 \exp(-210(0.00990)) = 10.3$$ 1
$$\neq 19$$

Thus $N = 19$ and $N_0 = 82$ does not satisfy (A)

If there is a constant background of k counts s^{-1} then (A) gives

$$(19-k) = (82-k)\exp[-210(0.00990)]$$ 1

Giving $$\frac{19-k}{82-k} = 0.125$$ 1

$$\underline{k = 10 \text{ counts } s^{-1}}$$ 1

[5]

(d) Here $N = N_1 e^{-\lambda_1 t} + N_2 e^{-\lambda_2 t}$ $\quad \lambda_2 \ll \lambda_1$ 1

half lives $T_2 \gg T_1$

For times $t \gg T_1$ the main source of radioactivity would be due to the second source. Then

$$N \simeq N_2 e^{-\lambda_2 t}$$ 1

Taking measurements and plotting $\ln N$ against t will give λ_2 and hence $T_2 = \ln 2/\lambda_2$. 1

[3]

Q8

a) $g = \dfrac{GM_E}{R_E^2}$ 1

$= \dfrac{(6.67\times10^{-11})(5.97\times10^{24})}{(0.638\times10^{7})^2} = G(1.4667\times10^{11})$ 1

$g = 9.78\ \mathrm{ms^{-2}}$ 1

[3]

b) 1ST SOLUTION

$g = \dfrac{GM}{R^2}$

$\Delta g = \dfrac{G\Delta M}{R^2} + (-2)\dfrac{GM\Delta R}{R^3}$ 1

$\dfrac{\Delta g}{g} = \dfrac{\Delta M}{M} - \dfrac{2\Delta R}{R}$ 1

Now

$\dfrac{\Delta R}{R} = -\dfrac{15\times10^3}{6.38\times10^6}$ and $\dfrac{\Delta M}{M} = \dfrac{-4\pi(6.38\times10^6)^2(15\times10^3)(2.7\times10^3)}{5.97\times10^{24}}$ 1

$= -2.35\times10^{-3}$ $= -3.47\times10^{-3}$ 1

Thus

$\dfrac{\Delta g}{g} = -3.47\times10^{-3} + 4.70\times10^{-3}$ 1

$\dfrac{\Delta g}{g} = +1.23\times10^{-3}$ (one of the marks for noting increase in g, +ve sign; one mark for numerical value) 2

7

2ND ALTERNATIVE SOLUTION

$g = \dfrac{GM_E}{R^2} = 1.4666\times10^{11}G$ (1)

At 15 km $g_{15} = \dfrac{G[5.97\times10^{24} - 4\pi(6.38\times10^6)^2(15\times10^3)(2.7\times10^3)]}{(6.38\times10^6 - 15\times10^3)^2}$ 2

$= \dfrac{G[5.97\times10^{24} - 2.0716\times10^{22}]}{(6.365\times10^6)^2}$

$= \dfrac{G\ 5.949\times10^{24}}{40.51\times10^{12}}$ 3

$= G(1.4684\times10^{11})$

From (1) $\Delta g = +G(1.8\times10^{8})$

$\dfrac{\Delta g}{g} = +\dfrac{1.8\times10^{-3}}{1.47} = +1.23\times10^{-3}$ 1 mark for +ve sign, increase in g; 1 mark for magnitude 2

[7]

Q8

c) $$\Delta g = \frac{G\ \frac{4}{3}\pi(2.5\times10^3)^3(7.9-2.7)10^3}{(2.5\times10^3)^2}$$ 3

$$= G\ \tfrac{4}{3}\pi\ (2.5\times10^3)(5.2)10^3$$
$$= G(5.4\times10^7)$$
$$= 6.67\times10^{-11}(5.4\times10^7)$$
$$= 3.63\times10^{-3}$$ $1\frac{1}{2}$

$$\frac{\Delta g}{g} = +3.71\times10^{-4}$$

magnitude 1 mark
sign +, increasing $\frac{1}{2}$ mark $1\frac{1}{2}$

[6]

d) If spherical volume of iron removed

$$\Delta g = -\frac{G(\frac{4}{3}\pi)(2.5\times10^3)^3\,7.9}{(2.5\times10^3)^2}$$ $1\frac{1}{2}$

$$= -G(\tfrac{4}{3}\pi)(2.5\times10^3)7.9$$
$$= -(6.67\times10^{-11})(\tfrac{4}{3}\pi)(2.5\times10^3)7.9$$ 1
$$= -5.52\times10^{-6}$$

$$\frac{\Delta g}{g} = -5.64\times10^{-7}$$

$\frac{1}{2}$ mark for -ve sign
1 mark magnitude $1\frac{1}{2}$

[4]

Q9

(a) (i)

$$\frac{GM_E}{(R_{EM}-R_{MF})^2} = \frac{GM_M}{R_{MF}^2} \qquad 1$$

$$\frac{R_{EM}-R_{MF}}{R_{MF}} = \left(\frac{M_E}{M_M}\right)^{\frac{1}{2}}$$

$$= \left(\frac{5.97\times10^{24}}{7.35\times10^{22}}\right)^{\frac{1}{2}} \qquad 1$$

$$= 9.01$$

$$R_{EM}-R_{MF} = 9.01\,R_{MF}$$

$$10.01\,R_{MF} = R_{EM} = 3.84\times10^{5}\text{ km}$$

$$\underline{R_{MF} = 3.84\times10^{4}\text{ km}} \qquad 1$$

(ii)

$$\frac{GM_E}{R_{EM}-R_{MP}} = \frac{GM_M}{R_{MP}} \qquad 1$$

$$\left.\begin{aligned}\frac{R_{EM}-R_{MP}}{R_{MP}} &= \frac{M_E}{M_M}\\ &= \frac{5.97\times10^{24}}{7.35\times10^{22}} = 8.12\times10\\ R_{EM} &= R_{MP}\,(82.2) = 3.84\times10^{5}\end{aligned}\right\} \quad 1$$

$$\underline{R_{MP} = 4.67\times10^{3}\text{ km}} \qquad 1$$

[6]

(b) Let v be the velocity of impact on lunar surface and M_R the mass of the rocket. At R_{MF} rocket has zero velocity if it has the minimum energy to enable it to reach the Moon. Conservation of energy requires

$$-GM_RM_M\left[\frac{1}{R_{MF}}\right] - GM_RM_E\left[\frac{1}{R_{EM}-R_{MF}}\right] = \frac{1}{2}M_Rv^2 - GM_RM_M\left[\frac{1}{R_M}\right] - GM_RM_E\left[\frac{1}{R_{EM}-R_M}\right] \qquad 4$$

$$\underline{v = \sqrt{2G\left[M_M\left(\frac{1}{R_M}-\frac{1}{R_{MF}}\right) + M_E\left(\frac{1}{R_{EM}-R_M}-\frac{1}{R_{EM}-R_{MF}}\right)\right]}} \qquad 1$$

5

Q9

(c) For a stable orbit radius r, velocity v_s, of satellite mass m_s (neglecting influence of Earth's gravitational field)

$$\frac{m_s v_s^2}{r} = \frac{G m_s M_M}{r^2}$$ 2

$$v_s = \left(\frac{G M_M}{r}\right)^{\frac{1}{2}}$$

$$= \left[\frac{(6.67\times10^{-11})(7.35\times10^{22})}{(10 + 1.74\times10^3)10^3}\right]^{\frac{1}{2}}$$ 1

$$= \left(\frac{4.90\times10^{12}}{1.75\times10^6}\right)^{1/2}$$

$$\underline{v_s = 1.67\ \mathrm{km\,s^{-1}}}$$ 1

[4]

(d) Neglecting the influence of the Earth's Gravitational field Orbital KE, T_0, is given by

$$T_0 = \tfrac{1}{2} m_s v_s^2 = \tfrac{1}{2}(1000)(1.67\times10^3)^2$$

$$\underline{T_0 = 1.40\times10^9\ \mathrm{J}}$$ 1

P.E. lost in descending to Moon's surface, that increases the KE,

$$V_D = G M_M m_s \left[\frac{1}{1.74\times10^6} - \frac{1}{1.75\times10^6}\right]$$ 1

$$= (6.67\times10^{-11})(7.35\times10^{22})(10^3)\left(\frac{10^4}{(1.74)(1.75)10^{12}}\right)$$ 1

$$= 4.90\times10^{15}\,(3.28\times10^{-9})$$

$$\underline{= 1.61\times10^7\ \mathrm{J}}$$ 1

Thus to 2 sig. figs energy to be extracted is

$$E_{min} = \underline{1.42\times10^9\ \mathrm{J}} \quad (i.e.\ 1.40\times10^9 + 1.61\times10^7\ \mathrm{J})$$ 1

[5]