1 (a) SOLUTION 2006 PAPER 2

(i) For the system of six cubes

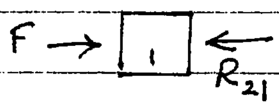
$$F = 6ma \qquad a = \text{acceleration} \quad [\tfrac{1}{2}]$$

$$a = \frac{F}{6m}$$

Acceleration of each cube and on system of six cube } is $\underline{a = \frac{F}{6m}}$ [½]

(ii) Resultant force on each cube $\underline{= ma = \frac{F}{6}}$ [½]

(iii) For cube 1,

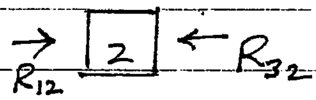


$$F - R_{21} = ma$$

$$R_{21} = F - ma = 6ma - ma$$

$$\underline{R_{21} = 5ma}$$

For cube 2,

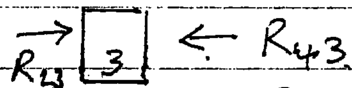


$$R_{12} - R_{32} = ma$$

$$R_{32} = -ma + R_{12}$$

$$= 4ma \qquad \text{as } R_{12} = R_{21}$$

For cube 3, $R_{23} - R_{43} = ma$

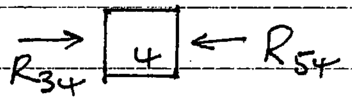


$$R_{43} = R_{23} - ma$$

$$= 4ma - ma \qquad \text{as } R_{23} = R_{32}$$

$$= 3ma$$

For cube 4, $R_{34} - R_{54} = ma$



$$R_{54} = R_{34} - ma$$

$$= 3ma - ma \qquad \text{as } R_{34} = R_{43}$$

[1½]

Force exerted on 5 by 4, $\underline{R_{45} = R_{54} = 2ma = \frac{1}{3}F}$ [1]

[[4]]

Alternatively starting from sixth cube

$$R_{56} = ma.$$

$$R_{45} - R_{65} = ma \qquad R_{45} = R_{54} = 2ma = \tfrac{1}{3}F$$

} [1½ + 1] = [2½]

Shorter: Consider together last two blocks

$$R_{45} = (2m)\frac{F}{6m} = \frac{F}{3}$$

} [1½]

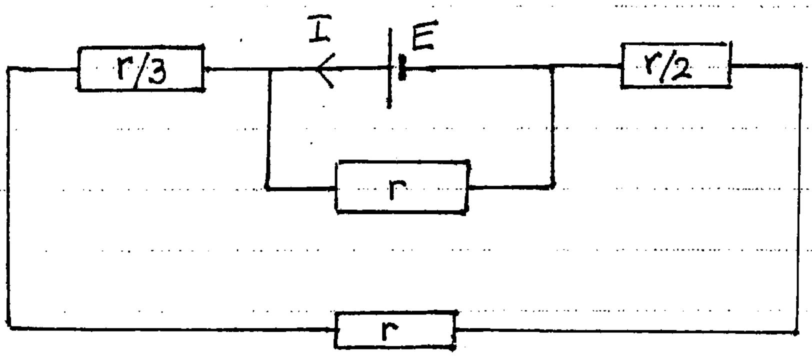
SOLUTION

1(b)

The total resis. of 3 resistors in parallel on LHS = $\left(\frac{1}{r}+\frac{1}{r}+\frac{1}{r}\right)^{-1} = \frac{r}{3}$ [$\frac{1}{2}$]

Total resis. of 2 resistors in parallel on RHS = $\left(\frac{1}{r}+\frac{1}{r}\right)^{-1} = \frac{r}{2}$ [$\frac{1}{2}$]

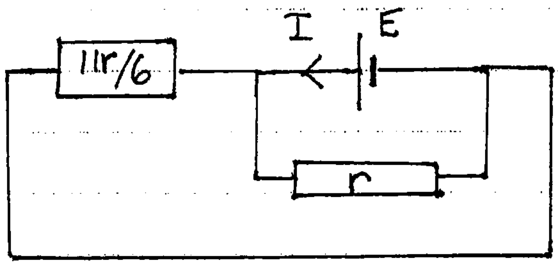
Circuit reduces to



[1]

$\left(\frac{r}{3}+\frac{r}{2}+r = \frac{11}{6}r \right)$

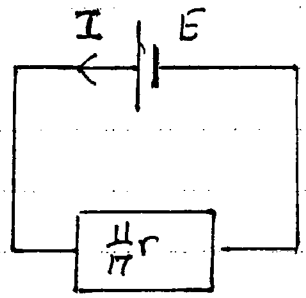
$\equiv$



[$\frac{1}{2}$]

$\frac{11}{6}r$ in parallel with r. Total resistance $\left(\frac{6}{11r}+\frac{1}{r}\right)^{-1} = \frac{11}{17}r$ [$\frac{1}{2}$]

$\equiv$



[$\frac{1}{2}$]

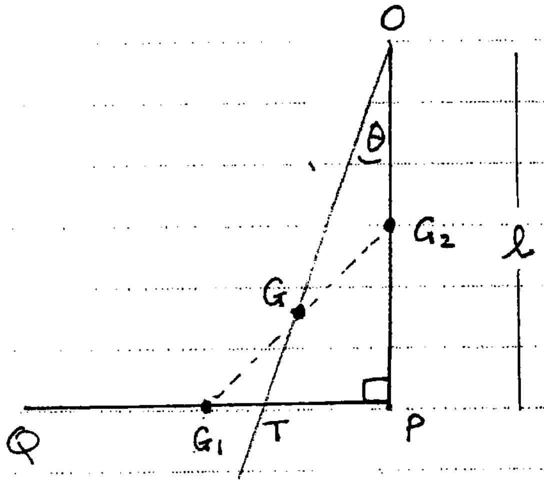
[$\frac{1}{2}$]

$I = \frac{17E}{11r}$

TOTAL [[4]]

CENTRE OF GRAVITY QUESTION

1(c)



(i) G mid way between G_1 and G_2, the centres of gravity of individual rods, i.e. mid way along each rod. Check measurement that G mid way between G_1 and G_2 at distance $0.35l$ from G_1 or G_2 ie $\frac{1}{4}l\sqrt{2}$ [1]

(ii) Angle between vertical OG and OP, θ, required.
θ can be obtained using trig. in right angled triangle OPT by measuring two sides of ΔOPT Method $\longrightarrow$ [1]

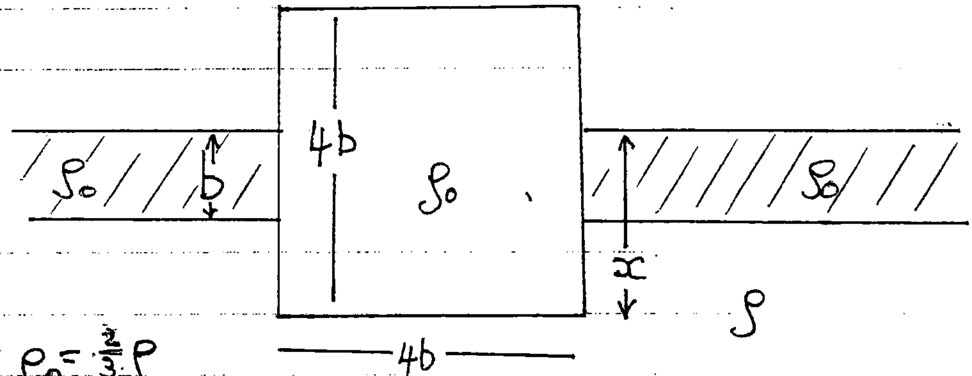
Correct measurements of the two sides and correct trig. formula [1]
Correct angle $\theta = 18.4^\circ \pm 0.4^\circ$ acceptable [1]

(Partial mark for angle in range $17.5^\circ - 17.9^\circ$ or $18.9^\circ - 19.3^\circ$ [$\frac{1}{2}$])

TOTAL [4]

Q1 (d)

SOLUTION



Let $\rho_0 = \frac{2}{3}\rho$

Let x be the height of the oil surface above the bottom of the stick.

Weight of stick $W = (4b \times 4b)\, l \rho_0 g$ where l length of stick [1/2]

Upthrust on stick $U = (4b)b\, l \rho_0 g + (x-b)(4b)\, l \rho g$ ① ① [2]

Equating, $W = U$, for equilibrium

$$(4b \times 4b)\, l \rho_0 g = (4b)b\, l \rho_0 g + (x-b)(4b)\, l \rho g$$ [1/2]

$$4b\rho_0 = b\rho_0 + (x-b)\rho$$

$$3b\rho_0 = (x-b)\rho$$ [1/2]

$$x = \frac{3b\rho_0}{\rho} + b$$ [1/2]

Fraction immersed F given by

$$F = \frac{x}{4b}$$ [1/2]

$$= \frac{3}{4}\frac{\rho_0}{\rho} + \frac{1}{4}$$

Substituting $\rho = \frac{2}{3}\rho_0$

$$F = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$ [1/2]

TOTAL [[5]]

SOLUTIONS

(i) The acceleration, a, produced by the Sun on the Earth and a mass in free fall is given by $a = R_{SE}\omega^2$ where ω is the angular frequency of the Earth about the Sun. Consequently, relative to the Earth, there is no net acceleration on the mass in free fall. Only one student produced this correct answer. Most students attempted to calculate a. We decided to give 3 marks for a correct calculation using the data provided (note: M_S not given) [4]

Calculation:
$$a = R_{SE}\omega^2 = \frac{4\pi^2}{T^2}R_{SE} \qquad [2]$$
$$= \frac{4\pi^2\,(1.5\times10^{11})}{(365\times24\times60\times60)^2} = 6.0\times10^{-3}\ \mathrm{ms^{-2}} \qquad [1]$$

(ii) Energy required for a man, mass m, to jump 1 m on Earth is mg J. [1]

Escape energy from a spherical body radius r, mass M, for man mass m is $\frac{GMm}{r}$ [1]

$$mgh = \frac{GMm}{r} \qquad [1]$$

$$gh = \frac{GM}{r} \qquad (A)$$

On Earth g given by

$$g = \frac{GM_E}{R_E^2} \qquad (B) \qquad [1]$$

From (A) and (B)

$$r = \frac{M}{M_E}\frac{R_E^2}{h} \qquad (C)$$

Sub^g $h = 1$m and $\frac{M}{M_E} = \frac{r^3}{R_E^3}$ in (C) assuming same uniform density

$$r^2 = R_E h = R_E \qquad \text{as } h = 1 \qquad [1]$$

Sub^g $R_E = 6.38\times10^6$m, $r = 2.53$km [1]

[[6]]

(iii) Rate of working W = (beats/sec)(vol. of blood/sec)(pressure) $[\frac{1}{2}+\frac{1}{2}+\frac{1}{2} = \frac{3}{2}]$

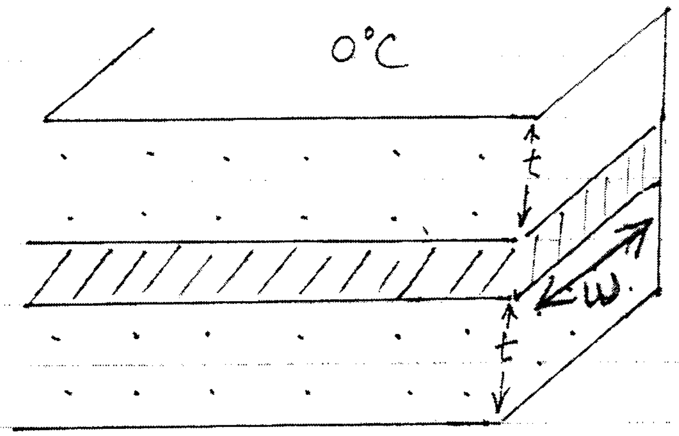
$$= \left(\frac{72}{60}\right)(7.5\times10^{-5})(19\times10^3)$$

$$= 1.7\ \mathrm{W} \qquad [\tfrac{1}{2}]$$

[[2]]

21 (f)

SOLUTIONS



~~(i) Rate of heat generated = I^2R~~ ~~[½]~~

(i) Consider the flow of heat from the copper strip through the insulating sheets. Heat, H, produced by current I, per unit length, is given by

(i) $$H = I^2R = I^2a(1+b\theta) \quad (A)$$ [1]

(ii) In a steady state, constant temp. gradient, from θ to 0°C. Heat conduction equation gives

$$H = 2KW\frac{\theta}{t} \quad (2 \text{ faces of strip}) \ (B)$$ [1]

Equating (A) and (B)

$$I^2a(1+b\theta) = 2KW\frac{\theta}{t}$$ [½]

$$1+b\theta = \frac{2KW}{I^2at}$$

$$\theta\left(\frac{2KW}{I^2at}-b\right) = 1$$

$$\underline{\theta = \left(\frac{2KW}{I^2at}-b\right)^{-1}}$$ [½]

Thus $\theta \to \infty$ when $\frac{2KW}{I^2at} \to b$

Critical current $I_c = \sqrt{\frac{2KW}{bat}}$ } [1]

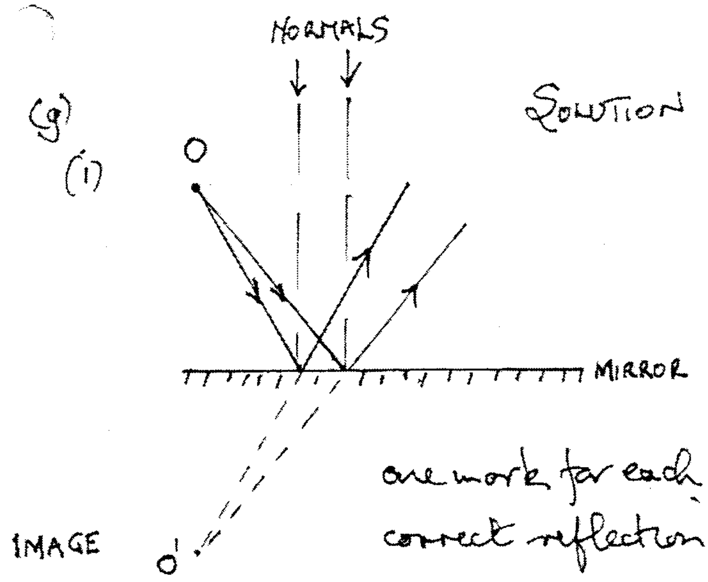
(iii) $$I_c = \sqrt{\frac{2(1.30\times10^{-1})(5.00\times10^{-3})}{(4.3\times10^{-3})(2.2\times10^{-2})(1\times10^{-3})}}$$ [½]

$$\underline{= 120\text{ A}}$$ [½]

[5]

21 (g) SOLUTION

(i)

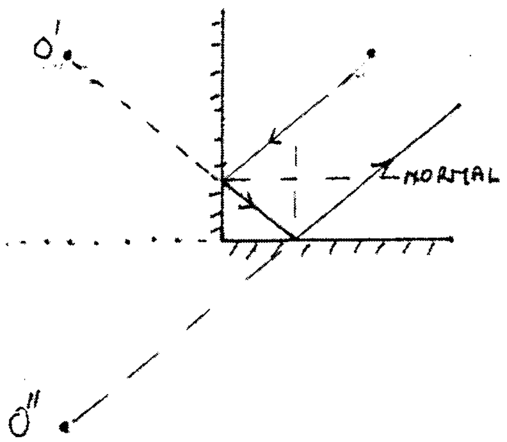


one mark for each correct reflection

TWO METHODS OF CONSTRUCTION

1. Virtual image at O' enables construction of reflected rays
2. Equal distance measurements each side of the normals enables construction of reflected ray with equal angles
(Examiner Check these equal distances) [2]

(ii)

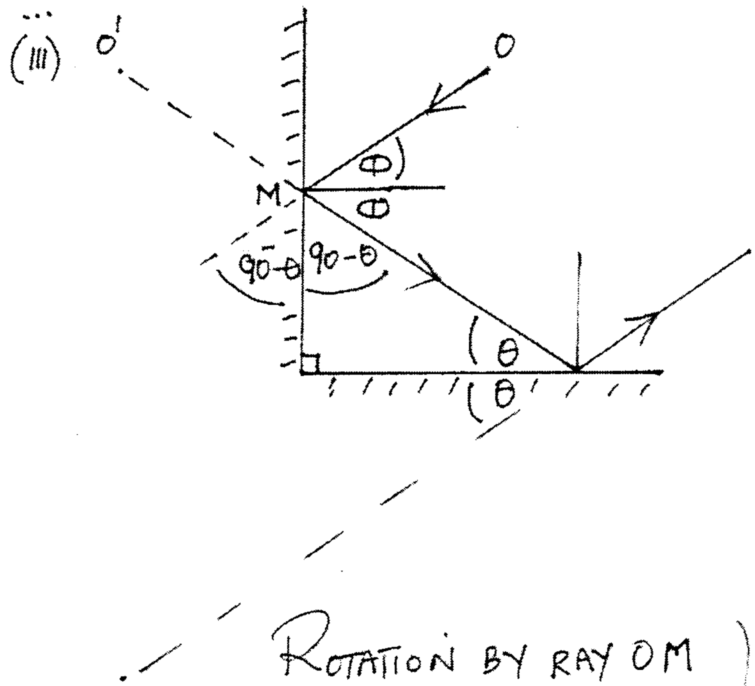


TWO METHODS OF CONSTRUCTION

1. VIRTUAL IMAGE METHOD AT O' and O''
2. CHECK EQUAL DISTANCE METHOD FROM NORMAL FOR BOTH REFLECTIONS [2]

One mark for each correct reflection

(iii)



Correct Diagram [1]

ROTATION BY RAY OM
$= 2(90 - \theta) + 2\theta$ [1]
$= 180°$ [1]

Proof required

[7]

21 (b)

SOLUTION

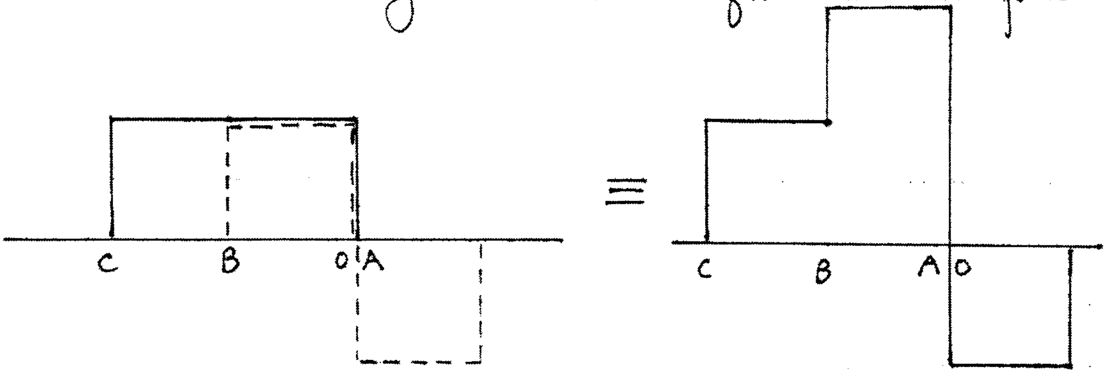
(i) Total intensity $= \underbrace{(1.0 \times 10^{-12})}_{\frac{1}{2}} \underbrace{4\pi (500)^2}_{1}$ W [1½]

$= \pi\ \mu W$

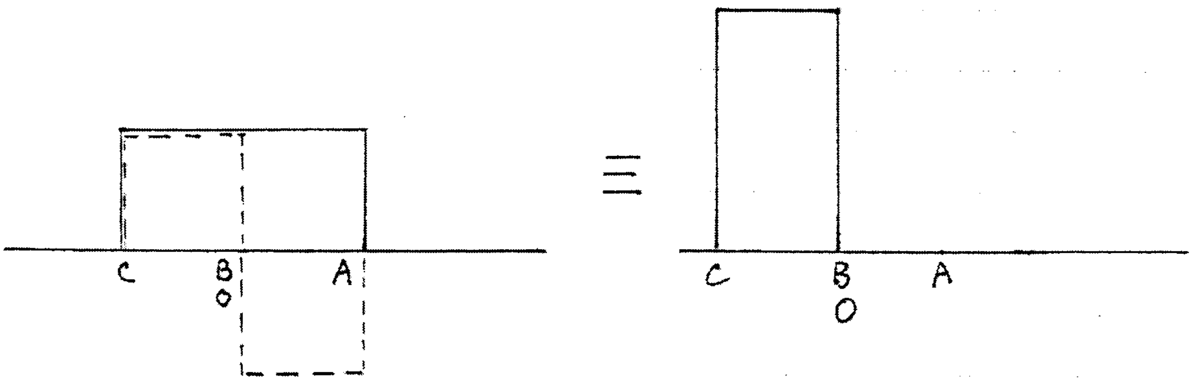
$= 3.14 \times 10^{-6}$ W $\Big\}\ \frac{1}{2}$ [½]

[[2]]

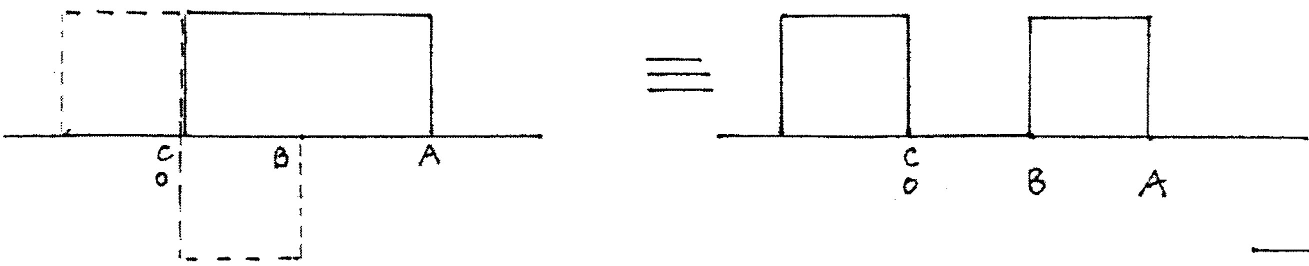
(ii) The resultant wave form is the sum of the two wave forms [1]



[1]



[1]

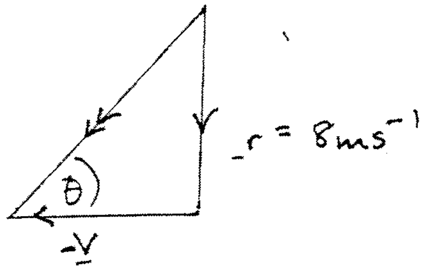


[1]

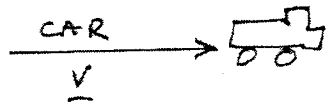
[[6]]

Q1 (j) SOLUTIONS

(i) Let car be travelling horizontally at $v\,ms^{-1}$. Then relative to the car the vertical velocity of the rain v_r is obtained by adding $\underline{v}_r$ and $(-\underline{v})$ [1]



CORRECT DIAG [1]



In diagram θ required angle

$$\tan\theta = \frac{v_r}{v} = \frac{8}{v}$$
$$\therefore \tan 30^\circ = \frac{8}{v}$$ [1]
$$v = 8\cot 30^\circ$$
$$\underline{v = 13.86\ ms^{-1}}$$ [½]

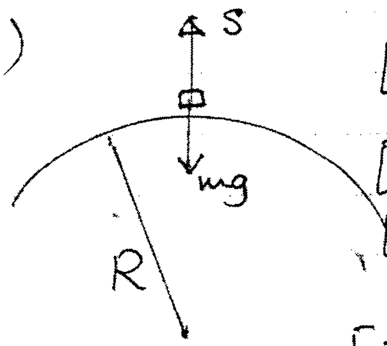
[[3½]]

(ii) If S is the reaction on the car

$$\frac{mv^2}{R} = mg - S$$ (Hump radius R) [1]

Max. v occurs when $S = 0$ [½]

$$v^2 = gR$$ [½]
$$= (9.81)\,40$$
$$\underline{v = 19.8ms^{-1}}$$ [½]



[[2½]]

[[6]]

(iii) Loss in PE ≡ Loss of P.E. in 25m of rope decending 20m

$$= \left[\left(\frac{25}{45}\right)15\right] g\ (20)$$ [1]
$$= \frac{500}{3}g$$ [½]

Using conservation of energy, with speed v,

" $\frac{1}{2}mv^2 = m'gh$ " where $m = 15kg$ and m' = mass of 25m r

$$\frac{1}{2}(15)v^2 = \frac{500}{3}g = \frac{500}{3}(9.81)$$ (A) [1]

$$\underline{\therefore v = 14.8\ ms^{-1}}$$ [½]

[ ]
[[4]]

Doubling the mass of rope does not change v [1] TOTAL [[10]]
as m appears on both sides of the equation (A)

Q1 K

SOLUTION

MASS DIFFERENCE $= (235.04 + 1.01) - (140.91 + 91.91 + 3 \times 1.01)u$
$= 0.20u$ (1)

Energy released $= 0.20 \times 931 = 186.20$ MeV [½]

235.04×10^{-3} kg of U^{235}_{92} contains 6.02×10^{23} atoms [½]

10 kg of U^{235}_{92} contains $\frac{10 \times 6.02 \times 10^{23}}{235.04 \times 10^{-3}}$ atoms [½]

$= 2.56 \times 10^{25}$ atoms [½]

Energy released by 10 kg of $U^{235}_{92} = 2.56 \times 10^{25} \times 186.2$ MeV
$= \underline{4.77 \times 10^{27} \text{ MeV}}$ [1]

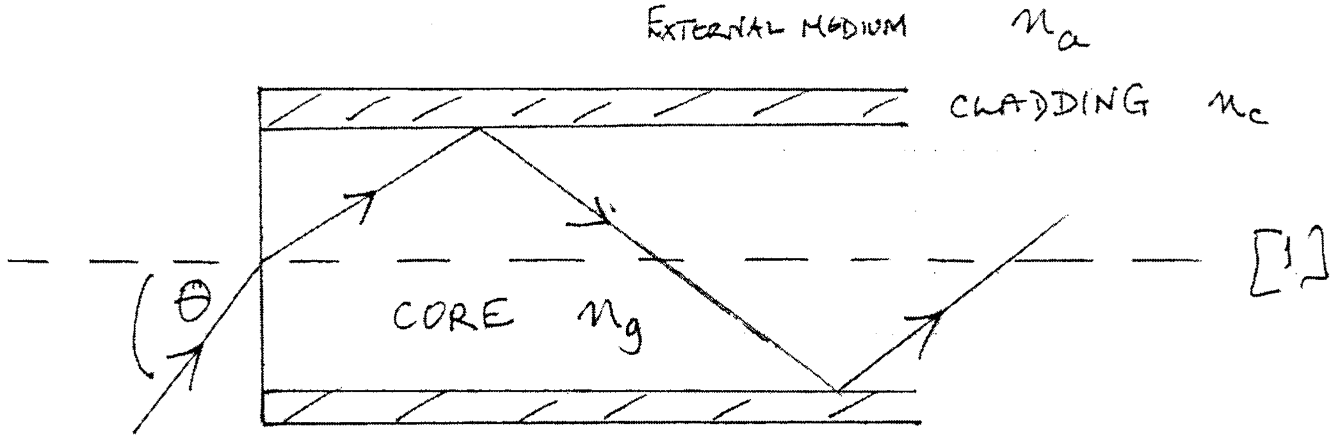
[[4]]

$1\text{eV} = 1.6 \times 10^{-19}$ J.

$4.77 \times 10^{27} \times 10^{6}$ eV
$= 4.77 \times 1.6 \times 10^{33-19}$ J.
$4.77 \times 1.6 \times 10^{14}$ J.

Q1(b)

SOLUTION



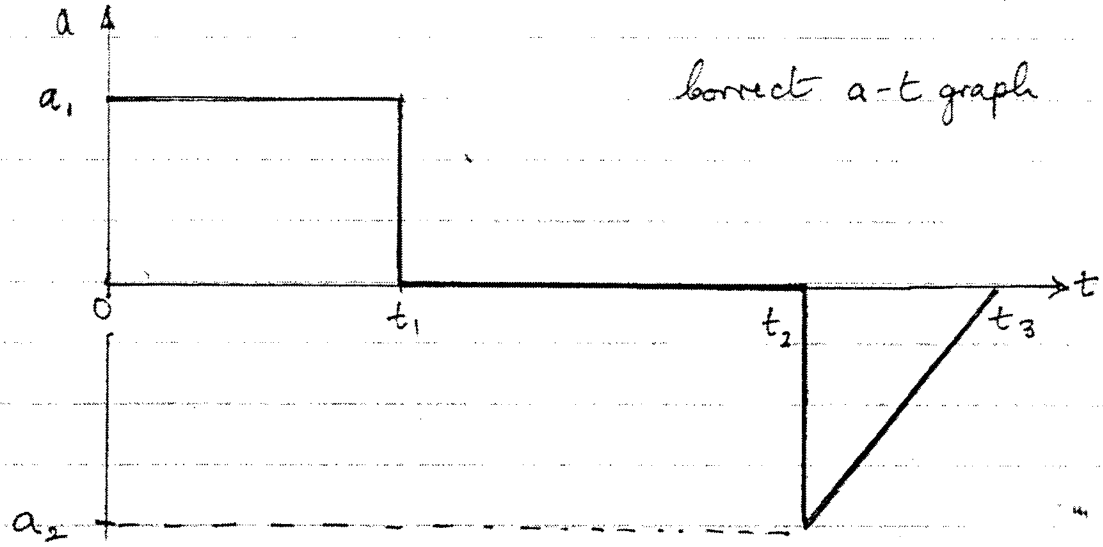
(i) Light is totally internally reflected at the interface between core and cladding ($n_c < n_g$) with negligable loss in intensity [1½]

(ii) If the angle of incidence at the (n_c-n_g) interface is less than that for total internal reflection, the intensity is rapidly diminished due to refraction through the cladding. This limits the angle of incidence, θ, at the fibre face. [1½]

[[4]]

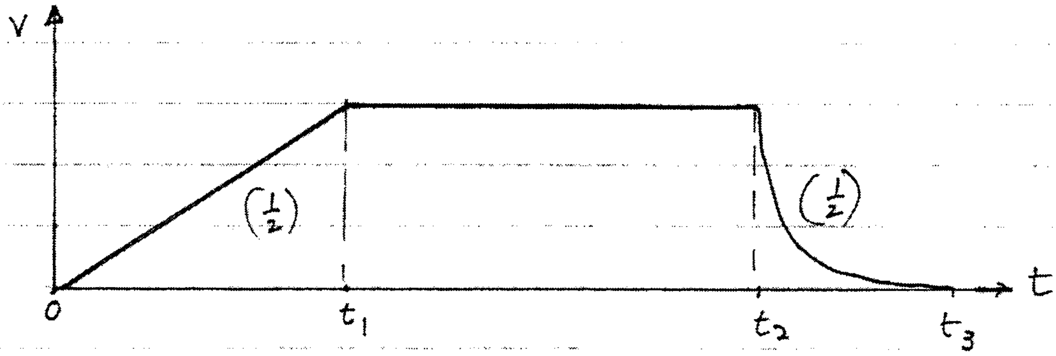
1 (m) SOLUTION 1 (m)

(i)



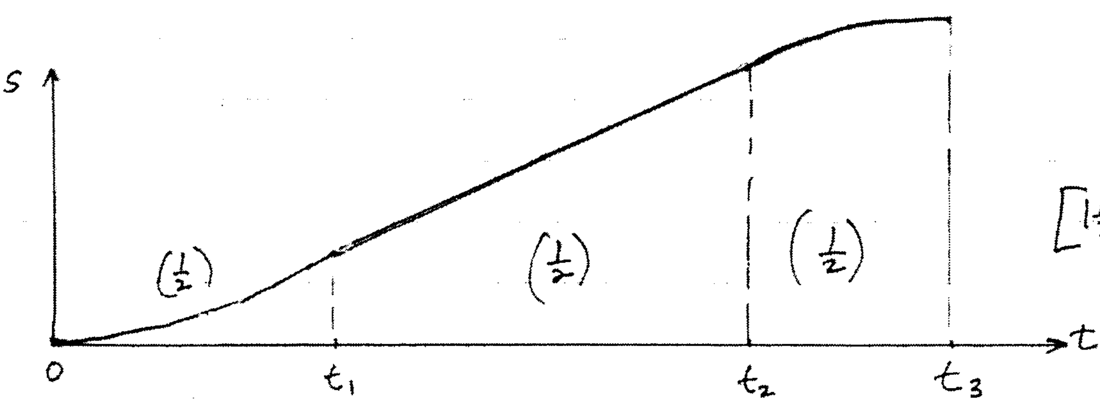
Correct a-t graph [$\frac{1}{2}$]

(ii)



[1]

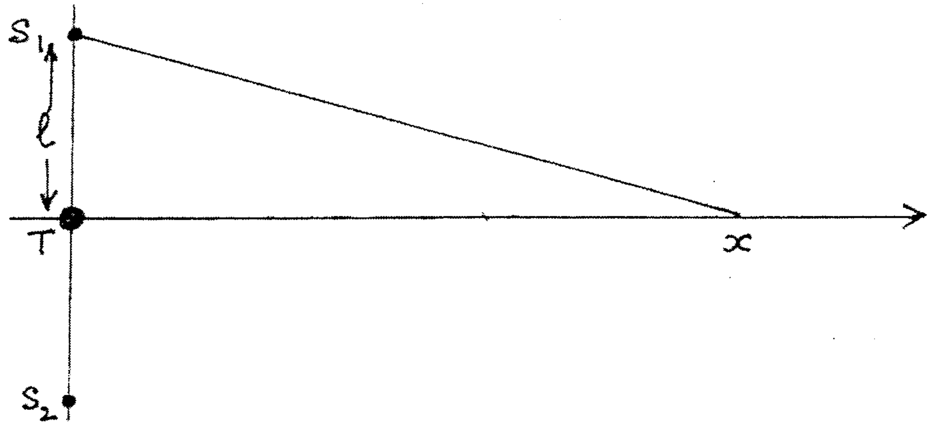
(iii)



[$1\frac{1}{2}$]

TOTAL [3]

Q2 SOLUTION



(a) (ii) Path difference

$$\phi = \sqrt{x^2+\ell^2} - x \quad [1]$$
$$= x\left[\left(1+\left(\tfrac{\ell}{x}\right)^2\right)^{\frac{1}{2}}-1\right] \quad [\tfrac{1}{2}]$$
$$= x\left[\tfrac{1}{2}\left(\tfrac{\ell}{x}\right)^2+\cdots\right] \quad [\tfrac{1}{2}]$$
$$= \frac{x}{2}\left(\frac{\ell}{x}\right)^2$$
$$= \frac{1}{2}\left(\frac{1}{4}\right)^2\frac{1}{x} \qquad \text{as } \ell=\tfrac{1}{4}$$
$$= \frac{1}{32}\,\frac{1}{x} \quad [1]$$

[$\frac{1}{2}$] + → [3]

(i) INSERT from top of next page $I \propto \frac{1}{r^2}$

(b) (i) Max occurs when $\frac{1}{32}\frac{1}{x} = (n+\frac{1}{2})\lambda$ or $\sqrt{x^2+\ell^2}-x = (n+\frac{1}{2})\lambda$ as S_1 and S_2 π out of phase with T [1]

(ii) Min. occurs when $\frac{1}{32}\frac{1}{x} = n\lambda$ or $\sqrt{x^2+\ell^2}-x = n\lambda$ [1]

Now $\lambda = \frac{c}{\nu} = \frac{330}{200\times10^3} = 1.65\times10^{-3}$ m [1]

(iii) Max. when $\frac{1}{32}\frac{1}{x} = (n+\frac{1}{2})\,1.65\times10^{-3}$

$n=0$ gives $\underline{x = 37.9\text{ m}}$ [1]
$n=1$ gives $\underline{x = 12.6\text{ m}}$ [1]
$n>1$ gives $x<10$ not required as $x\geq10$ [$\frac{1}{2}$]

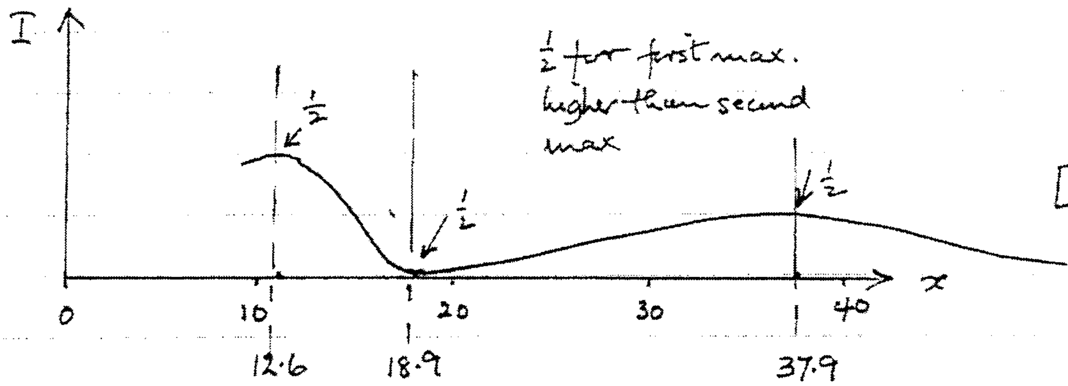
Min. when $\frac{1}{32}\frac{1}{x} = n\,(1.65\times10^{-3})$

$n=1$ gives $\underline{x = 18.9\text{ m}}$ [1]
$n>1$ gives $x<10$ [$\frac{1}{2}$]

↑ meet with after (a)(i)

2 (iv) $I \propto \frac{1}{r^2}$ [½]

(b) (iv)



[2]

[[13]]

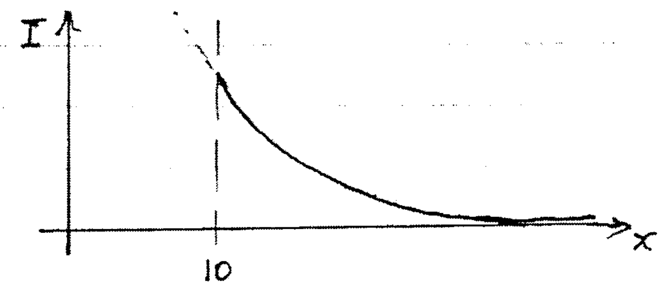
(c) Intensity $I = kA^2$ where A amplitude and k a constant

(i) T has 4 times power of S_1 or S_2
T has 2 times the amp. of S_1 or S_2, say 2α
Total amp. at interference maxima is 4α
So intensity is $k16\alpha^2$; 16 times that of S_1 or S_2 alone
If S_1 and S_2 switched off amp. reduces to 2α, intensity $4\alpha^2$ ie <u>power falls by a factor of 4</u> [2]

(ii) Similarly if T switched off, total amp 2α and intensity $k4\alpha^2$; reduced from $k16\alpha^2$
<u>Power thus falls by factor of 4</u> [2]

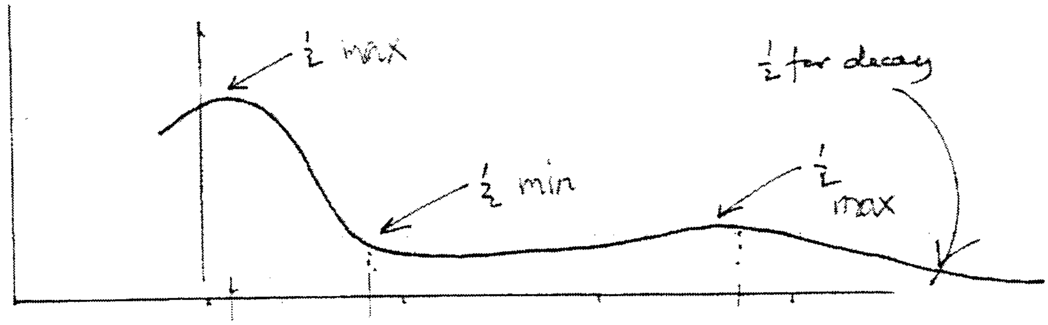
[[4]]

(d) (i) $A = \frac{2K}{x}$ S_1 and S_2 in phase, K const [1]
$I = k\frac{4K^2}{x^2}$



[1]

(d)(ii) S_2 switched off
Interference occurs, ∴ S_2 and T π out of phase, but minimum does not have zero intensity



[2]

[[4]]

TOTAL [[[20]]]

Q3 SOLUTION

(a)(i) $i_{2,2}$ is i_2 when 2V short circuited

$i_{2,6}$ " " " 6V " "

$i_{1,2}$ is i_1 " 2V " "

$i_{1,6}$ is i_1 " 6V " "

$$i_{2,6} = \frac{2.0}{6+(\frac{1}{2}+\frac{1}{4})^{-1}} = \frac{2.0}{6+4/3} = \frac{6}{22} = \frac{3}{11} \text{ amps} \quad [1]$$

$$\therefore i_{1,6} = -\frac{4}{6}\left(\frac{6}{22}\right) = -\frac{2}{11} \text{ amps} \quad [1]$$

(ii) $$i_{1,2} = \frac{6.0}{2+(\frac{1}{4}+\frac{1}{6})^{-1}} = \frac{6.0}{2+(5/12)^{-1}} = \frac{5(6)}{22} = \frac{15}{11} \text{ amps} \quad [1]$$

$$\therefore i_{2,2} = -\frac{4}{10}\left(\frac{15}{11}\right) = -\frac{6}{11} \text{ amps} \quad [1]$$

[[4]]

(b)(i) $$i_1 = i_{12} + i_{16} = -\frac{2}{11} + \frac{15}{11} = \frac{13}{11} = 1.18 \text{ amps} \quad [1]$$

$$i_2 = i_{22} + i_{26} = \frac{3}{11} - \frac{6}{11} = -\frac{3}{11} = -0.27 \text{ amps} \quad [1]$$

(ii) For 4Ω $$i_4 = \frac{6}{10}\left(\frac{15}{11}\right) + \frac{2}{6}\left(\frac{3}{11}\right) = \frac{10}{11} = 0.91 \text{ amps} \quad [1]$$

Alternatively $i_4 = i_1 + i_2 = 1.18 - 0.27 = .91$ in downward direction

(iii) KIRCHOFF'S EQUNS.

LHS loop $\quad 6 = 2i_1 + 4(i_1+i_2)$ (A) [1]

RHS Loop $\quad 2 = 6i_2 + 4(i_1+i_2)$ (B) [1]

From (b)(i) we also have

$$i_1 + i_2 = \frac{10}{11}$$

Sub^g into RHS of A $\quad 2\left(\frac{13}{11}\right) + 4\left(\frac{10}{11}\right) = \frac{66}{11} = 6 = \text{LHS}$ [1]

Sub^g into RHS of B $\quad 6\left(-\frac{3}{11}\right) + 4\left(\frac{10}{11}\right) = \frac{22}{11} = 2 = \text{LHS}$ [1]

Thus Kirchoff's eqns. verified

[[7]]

(c)(i) Power developed in 4Ω $$= \left(\frac{10}{11}\right)^2 4 = \frac{400}{121} = 3.31 \text{ W} \quad [1]$$

(ii) Energy conversion $$\left(\frac{13}{11}\right)6 = \frac{78}{11} = 7.09 \text{ W} \quad [1]$$

[[2]]

(d)(i) The voltages are replaced by

$2\cos\omega t$ V for 2 V

$6\cos\omega t$ V for 6 V

} [1]

Giving $\quad i_{22} = \frac{3}{11}\cos\omega t \qquad i_{12} = -\left(\frac{1}{11}\right)\cos\omega t$ [1]

$\quad i_{16} = \frac{15}{11}\cos\omega t \qquad i_{26} = -\left(\frac{6}{11}\right)\cos\omega t$ [1]

Alternative forms with cos replaced by $\sin(\omega t+\phi)$ or $\cos(\omega t+\phi)$, for any ϕ, acceptable ~~here and in~~ (d)

(d)(i) $\quad i_1 = -\frac{2}{11}\cos\omega t + \frac{15}{11}\cos\omega t = \frac{13}{11}\cos\omega t$ [1]

$\quad i_2 = \frac{3}{11}\cos\omega t + \left(-\frac{6}{11}\right)\cos\omega t = -\frac{3}{11}\cos\omega t$ [1]

(ii) $\cos\omega t$ replaced by $\cos(\omega t+\pi)$ in one of the voltages [1]

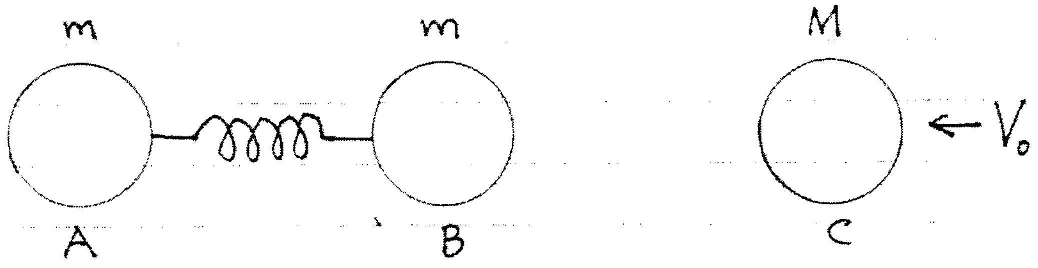
i.e. $\cos\omega t$ replaced by $-\cos\omega t$ [1]

any equivalent statement OK eg $(\omega t-\pi)$ with sin/cos functions

[[7]]

TOTAL [[20]]

Q4 SOLUTIONS



(a) (i) Energy Eqn. $\frac{1}{2}MV_0^2 = \frac{1}{2}mV_B^2 + \frac{1}{2}MV_c^2$ (A) [1]

MOMENTUM EQN. $MV_0 = mV_B + MV_c$ (B) [1]

(ii) FIRST SOLUTION

Sub^g $V_A = V_B = 0$ and $V_c = V_0$ in eqn. (A)

$LHS = \frac{1}{2}MV_0^2$, $RHS = \frac{1}{2}MV_0^2$, HENCE EQUN. (A) SATISFIED [½]

Sub^g $V_A = V_B = 0$ and $V_c = V_0$ in (B)

$LHS = MV_0$, $RHS = MV_0$, HENCE EQUN. (B) SATISFIED [½]

SECOND SOLUTION

Sub^g $V_c = \frac{M-m}{M+m}V_0$, $V_B = \frac{2M}{M+m}V_0$ and $V_A = 0$ in (A)

$$LHS = \frac{1}{2}MV_0^2$$

$$\begin{aligned} RHS &= \frac{1}{2}m\left(\frac{2M}{M+m}\right)^2 V_0^2 + \frac{1}{2}M\left(\frac{M-m}{M+m}\right)^2 V_0^2 \\ &= \frac{1}{2}\frac{MV_0^2}{(M+m)^2}\left[4mM + M^2 - 2mM + m^2\right] \\ &= \frac{1}{2}\frac{MV_0^2}{(M+m)^2}\left[M^2 + 2mM + m^2\right] \\ &= \frac{1}{2}\frac{MV_0^2}{(M+m)^2}(M+m)^2 \\ &= \frac{1}{2}MV_0^2 = LHS \end{aligned}$$ [3]

EQUATION VERIFIED

Sub^g into (B) $RHS = m\left(\frac{2M}{M+m}\right)V_0 + M\left(\frac{M-m}{M+m}\right)V_0 = MV_0 = LHS$ EQN VERIFIED [1]

(iii) $V_A = 0$ as after collision only balls to experience change in velocity are B and C. Spring does not communicate a change in momentum to A initially [1]

First solution rejected as it does not correspond to a collision – velocities of all masses remain unchanged [1

(iv) C. of M. moves with average velocity of A and B as masses equal

$$V_{cM} = \left(\frac{M}{M+m}\right)V_0$$ (in direction of $\underline{V_0}$) [1]

[[10]]

(b) (i) Spring ~~vibrates~~ about C. of M of A and B

k' spring constant for half the spring

If full spring extended by $2x$, half spring extended by x, so tension T in spring given by

$$T = 2kx \quad \text{for full spring}$$

But $$T = k'x \quad \text{for half spring}$$

$$\therefore \underline{k' = 2k}$$

[2]

So angular frequency of half spring $\omega = \sqrt{\frac{k'}{m}}$ is given by

$$\underline{\omega = \sqrt{\frac{2k}{m}}}$$

[1]

(ii) Energy conservation in C. of M. system, for masses on spring, each mass has max speed of $\left(\frac{M}{M+m}\right)V_0$;

[2]

$$2\left(\tfrac{1}{2}m\right)\left(\frac{M}{M+m}V_0\right)^2 = 2\left(\tfrac{1}{2}k'A^2\right)$$

$$\frac{mM^2V_0^2}{(M+m)^2} = 2\left(\tfrac{1}{2}(2k)A^2\right) \quad \text{as } k' = 2k$$

$$A^2 = \frac{m}{2k}\left(\frac{M}{M+m}\right)^2 V_0^2$$

$$\underline{A = \sqrt{\frac{m}{2k}}\left(\frac{M}{M+m}\right)V_0}$$

[2]

OR $$A = \frac{V_0}{\omega(1+\gamma)} \quad \text{where } \gamma = \frac{m}{M}$$

(iii) $$x_B = V_{cm}t + A\sin\omega t$$ [2]

$$\underline{= \frac{V_0}{1+\gamma}t + \frac{V_0}{(1+\gamma)\omega}\sin\omega t}$$ [1]

where $\gamma = \frac{m}{M}$ and $\omega = \sqrt{\frac{2k}{m}}$

[[10]]

Q5

SOLUTION

a) (i) $I=4$, $V=4428$ too large an increment from $I=3$ to $I=4$ compared with that produced by other integer changes which exclude $I=4$ [1]

(ii) Last but one digit "2" probably mistyped should be "0" ie $V=4408$ correct reading [1]
(no marks for just stating "error")
For answer "ignore reading" give [½] in place of [1])

(iii) Plot graph of $R=\frac{V}{I}$ against I^2
Straight line graph with acceptable scale using full [2]
page of graph paper for range of values in the table. [1]
Correct table of values [1]

(iv) $$R = \lambda I^2 + R_0$$
Obtain from graph
R_0 value (intercept) : 1099.80 [1]
Error in R_0 ± 0.05 acceptable [1]
λ value (gradient) 0.120 [1]
Error in λ ± 0.002 acceptable [1]

Formula: $R = 0.120\, I^2 + 1099.80$ [1]

[11]

(b)(i)
$$E = \rho R^{\alpha} t^{-2} \qquad \text{(A)}$$
$$\ln(E) = \ln(\rho) + \alpha \ln(R) - 2\ln(t)$$
$$\ln(t) = \tfrac{1}{2}(\ln(\rho) - \ln(E)) + \tfrac{1}{2}\alpha \ln(R)$$
$$\ln(t) = \tfrac{1}{2}\ln\left(\tfrac{\rho}{E}\right) + \tfrac{1}{2}\alpha \ln(R)$$
[2]

Plot $\ln(t)$ against $\ln(R)$, it should be a straight line gradient $\frac{1}{2}\alpha$ intercept $\frac{1}{2}\ln(\rho/E)$ [2]

(b)(ii) Let $\frac{1}{2}\ln\left(\rho/E\right) = k$, the intercept along the $\ln(t)$ axis where $\ln(R)=0$. [1]

Then

$$\frac{\rho}{E} = e^{2k}$$

$$\underline{\rho = Ee^{2k}}$$ [1]

(iii) Plotting $\ln(t)$ against $\ln(R)$ gives a gradient of $\left(\frac{\alpha}{2}\right)$ from which α can be obtained [1]

(iv) Dimensions / units of equation (A) gives

$$ML^2T^{-2} = ML^{-3}L^{\alpha}T^{-2}$$ [1]

Thus equating powers of L,

$$\underline{\alpha = 5}$$ [1]

[9]

Q6

SOLUTION

a (i) Area of annulus $= \pi (n+1)^2 t^2 - \pi n^2 t^2$

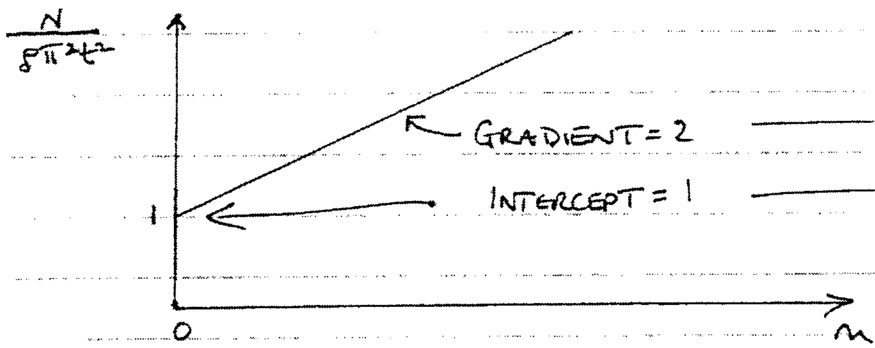
$= \pi t^2 [\, n^2 + 2n + 1 - n^2 \,]$

$= \pi t^2 (2n+1)$ [1]

Number of particles in annulus

$\underline{N = \rho \pi t^2 (2n+1) V}$ [1]

(ii)

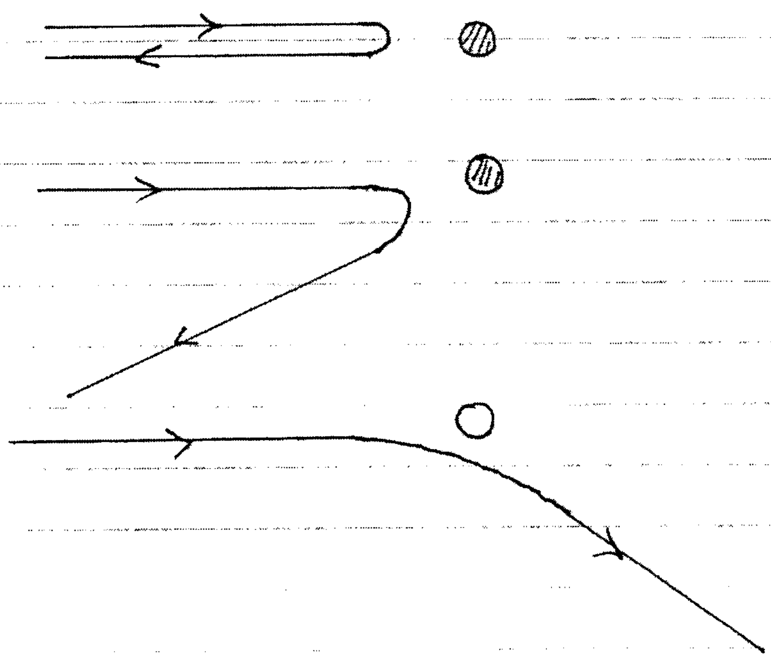


[½]

[½]

[[3]]

(b) (i)



[1]

[½]

[½]

(ii) Small angles associated with small n and hence small N ie few particles directly incident on the nucleus [1]

(iii) π^c or $180°$ [1]

(iv) Electron mass a factor 8000 smaller than α particle ie exceedingly small relative inertia [1]

[[5]]

charge on α particle 2e not e.

(c)(i) Conservation of energy requires

$$\tfrac{1}{2}4mu^2 = \frac{Ze^2}{(4\pi\varepsilon_0)r_1} = \frac{(2)79e^2}{(4\pi\varepsilon_0)r_1} \qquad [2]$$

$$r_1 = \frac{2(79)e^2}{(2)(4\pi\varepsilon_0)mu^2}$$

$$\underline{r_1 = \frac{79e^2}{(4\pi\varepsilon_0)mu^2}} = \qquad [1]$$

(ii) Zero relative velocity at distance of closest approach [1]
ie both particles have same speed, v
α particle has mass 4m

Conservation of energy gives

$$\frac{1}{2}(4mu^2) = \overset{(1)}{\frac{1}{2}(4mv^2)} + \overset{(1)}{\frac{1}{2}(197mv^2)} + \overset{(1)}{\frac{(2)79e^2}{(4\pi\varepsilon_0)r_2}} \quad (A) \qquad [3]$$

Conservation of momentum gives

$$4mu = 4mv + 197mv \qquad [2]$$

$$4u = 201v$$

$$\underline{v = \frac{4u}{201}} \qquad [1]$$

Substituting $v = \frac{4u}{201}$ into (A)

$$4u^2 = 201\left(\frac{4u}{201}\right)^2 + \frac{(2)158e^2}{(4\pi\varepsilon_0)mr_2} \qquad [1]$$

(c)(ii)

$$\frac{(804-16)u^2}{201} = \frac{(2)158e^2}{(4\pi\varepsilon_0)mr_2}$$

$$r_2 = \frac{(2)(158)(201)e^2}{(4\pi\varepsilon_0)(788)mu^2} \qquad *$$

$$= \underline{\frac{(2)15{,}879e^2}{394(4\pi\varepsilon_0)mu^2}} = \underline{\frac{31758e^2}{394(4\pi\varepsilon_0)}} \qquad [1]$$

[[12]]

* (NOTE $\frac{201}{788} \sim \frac{1}{4}$, compare result (c)(i))

TOTAL [[22]]

Q7

SOLUTION

(a)(i) Using "$v^2 = u^2 - 2gh$" where $v = 0$ and $u = v$ in formula quoted

$$0 = v^2 - 2gH$$

$$\underline{H = \frac{v^2}{2g}}$$ [1]

(ii) Using "$s = ut - \frac{1}{2}gt^2$"

$$0 = vt - \frac{1}{2}gt^2$$

$$\underline{t = \frac{2v}{g}} \quad \text{(A)}$$ [1]

[[2]]

(b)(i) Resultant force $mg\sqrt{1+\alpha^2}$ [$\frac{1}{2}$]
in direction $\theta = \tan^{-1}\left(\frac{1}{\alpha}\right)$ [$\frac{1}{2}$]

(b) (ii)

Consider horizontal motion. A distance s horizontally is given by

$$s = ut + \frac{1}{2}(\alpha g)t^2 \quad \text{(B)}$$ [1]

From vertical motion, returning to ground in time t, using (A)

$$t = \frac{2v}{g} \quad \text{(C)}$$ [1]

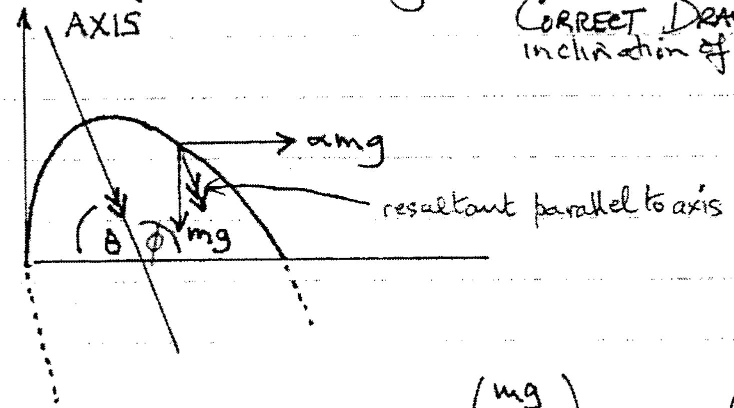
$$\therefore \text{Range} \quad R = u\left(\frac{2v}{g}\right) + \frac{1}{2}(\alpha g)\left(\frac{2v}{g}\right)^2$$ Sub^n. (C) into (B)

$$\underline{R = \frac{2v}{g}[u + \alpha v]} \quad \text{(D)}$$ [1]

(iii) Parabola [1]

Axis along direction of resultant force (see diagram) CORRECT DIRN [1]

CORRECT DRAWING and inclination of axis [1]



Axis of symmetry at angle $\theta = \tan^{-1}\left(\frac{mg}{\alpha mg}\right) = \tan^{-1}\left(\frac{1}{\alpha}\right)$ [1]

(As in case of no wind, $\alpha = 0$, axis along direction of mg)

(iv)

(b) (iii) Resultant force on m must be in the opposite direction of initial velocity (viz with $\alpha = 0$ this direction is vertically upwards — opposite to g); u must be negative (in -ve x direction) so that

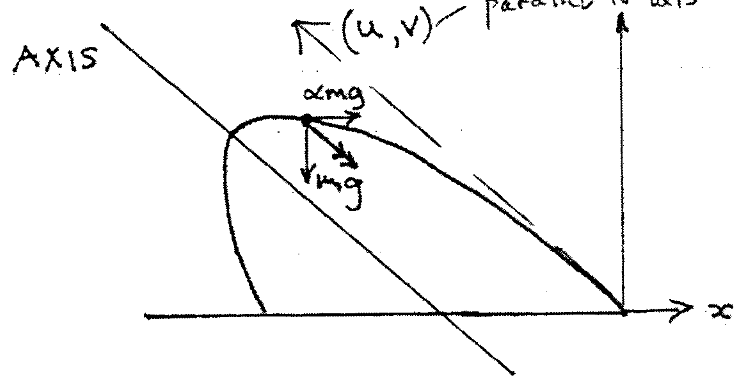
$$\frac{u}{v} = -\frac{\alpha mg}{mg} = -\alpha \quad \text{or} \quad \frac{v}{u} = -\frac{1}{\alpha}$$ [2]

$$\tan\theta = +\frac{1}{\alpha}$$

$\tan^{-1}$ correction

$$\theta = \tan^{-1}\left(\frac{1}{\alpha}\right) = \frac{-v}{u}$$ where u -ve [1]

Alternatively $\phi = \tan^{-1}\left(-\frac{1}{\alpha}\right) = \frac{v}{u}$ (note u -ve) See diagram for θ and ϕ on previous page
$= 180 - \theta$



[[11]]

(c) (i) From result (a)(i) maximum height given by "$H = \frac{v^2}{2g}$" when "g" resultant force. When "g" replaced by $g\sqrt{1+\alpha^2}$ at angle θ due to resultant force of wind and gravity, the distance travelled along the inclined straight path is

$$H = \frac{\sqrt{v^2+u^2}}{2g\sqrt{1+\alpha^2}}$$ [1]

as "v" replaced by $\sqrt{v^2+u^2}$ for inclined path where mass m experiences a constant force $mg\sqrt{1+\alpha^2}$. [1]

Work done W = Const force × distance
$= -mg\sqrt{1+\alpha^2}\left(\sqrt{v^2+u^2}/2g\sqrt{1+\alpha^2}\right)$
$= -\frac{1}{2}m(v^2+u^2) = -\frac{1}{2}mv^2(1+\alpha^2)$ (as $u = -\alpha v$) [1]

Alternatively using conservation of energy initial k.e., $\frac{1}{2}m(v^2+u^2)$ converted into work done by mass / pe on reaching max. height along straight path giving

Work done on mass $= W = -\frac{1}{2}m(u^2+v^2) = -\frac{1}{2}mv^2(1+\alpha^2)$ ([3])

1 (c) (ii) For a conservative potential field, work done indep. [1]
of path. So as one can reach final position by [1]
following a horizontal path

$$W = \alpha m g R$$ [1]

$$W = \alpha m g \frac{2v}{g}(u+\alpha v)$$ ~~from~~ (D) [1]

$$\underline{W = 2\alpha m v(u+\alpha v)}$$

[[7]]

TOTAL [[[20]]]